# The role of externalities in determining the average level of corruption in the production process

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## Abstract

A model of corruption in the production process is built which is aimed at studying the problem of choosing the optimal scheme of punishment, taking into account the structure of given production relations, as well as the fixed budget assumption. Stable equilibria of the model are characterized, and the mechanism of a bureaucrat's decision-making process is described. The crucial role of strategic interactions among bureaucrats is emphasized. Then, the model is used to examine punishment schemes: to what degree should punishment schemes depend on the deviation from the average level of corruption by a given agent. Normally, it is better for a policymaker to have a scheme which reacts sharply to the deviations.

# NON-TECHNICAL SUMMARY

This paper is devoted to the analysis of corrupt relations in the economy. My aim is to adequately characterize such relations by specifying the role of alternative factors of corruption and their interactions, as well as to study the impact of alternative anti-corruption measures on the average level of corruption in the economy. A particularly interesting question is the efficiency of one special measure, namely, the redistribution of resources devoted to fighting corruption from highly corrupt agents to moderately corrupt ones or vice versa.

In order to achieve these goals, an economic model is constructed and studied. The main assumptions of the model are:

- a sufficiently large number of agents (bureaucrats);
- a monopoly position of any bureaucrat in providing illegal services to a number of firms under his jurisdiction;
- the level of illegal activity, which is the degree of a bureaucrat's involvement in corruption can take any real value;
- the presence of a central authority whose goal is to reduce the average level of corruption as much as possible; this authority possesses a certain budget and is free to choose one or another *scheme of punishment* out of an exogenous set of such schemes.

Bureaucrats make their decisions about how much to involve themselves in corruption by comparing their private benefits from doing so with the costs. Benefits are higher when the degree of involvement is higher, but costs also rise. This cost-benefit trade-off is of a rather complex nature because of the existence of mutually re-enforcing influences among bureaucrats: costs are lower when the overall level of corruption goes up because the earmarked budget for fighting corruption is scarce. This is a crucial point of the paper. We analyze the nature of this trade-off by assuming that costs of being corrupt depend on two variables: the individual level of involvement, and the average level corruption. The central authority can affect to a certain degree the reciprocal influences among bureaucrats through redistributing its available budget. This redistribution determines how private costs of being corrupt depend on the overall level of corruption.

This model provides the following insights:

- Society can find itself in two long-lasting states. The first one is corruption-free, and the second one is characterized by a high average level of corruption. This study is mainly concerned with the second type of equilibrium.

- The effects of a particular policy decision are accumulating in time. In other words, the long-run consequences of a change in policy are stronger than the short-run ones, and their direction is the same.

As for the main question of the paper — How do mutual influences between bureaucrats affect the level of corruption? — requires distinguishing between the following two cases:

(1) When there are no fixed costs of corruption, the differentiation in bureaucrats' choices rises if costs weakly depend on the overall level; probably the average corruption level increases as well.

(2) When fixed costs are present, the weaker dependence of costs on the average level of corruption results in all the corrupted bureaucrats increasing their levels of involvement. Otherwise, they simply refrain from corruption at all; in addition, either the differentiation in agents' choices, or the average level of corruption increases (maybe even both of them).

The main conclusion of the paper is that reducing the role of reciprocal influence among bureaucrats is likely to result in a rise in the average level of corruption. Hence, such a policy may not be desirable. A by-product of reducing reciprocal influences is a higher differentiation in individual corruption levels.

# 1 Introduction

## 1.1 Corruption today: an overview

The problem of corruption is definitely one of the most pronounced in the current world. Many countries (e.g., Zaire, Kenga, Nigeria, some other African states, and probably also India, Indonesia (see Shleifer and Vishny (1993)) have a corruption level so high that losses are comparable to their GDP (this is also mentioned in Satarov, Levine and Tsirik (1998), Bardhan (1997)). Most peoples in Africa, Latin America and East Asia live at about the survival level, and most scientists agree that total corruption is one of the primary reasons for that (see Shleifer and Vishny (1993), Satarov, Levine and Tsirik (1998), Rose–Ackerman (1978), as well as many other investigations). In recent years some countries in transition joined this cohort of corrupt states, and Russia is undoubtedly among the leaders, if not the very first one. According to Satarov, Levine and Tsirik (1998), losses from corruption in Russia are high enough to justify any thinkable expenditure to fight it.

At the same time, one would probably suggest that in a centrally planned economy, corruption may bring some benefits. Alternative studies confirm this view, e.g., Ericson (1983) convinces readers that corruption is necessary (as a part of a general *shadow economy*) to facilitate a fairly well-functioning planned economy, such as the former Soviet one. In fact, the presence of corruption is a Pareto-improvement over an imaginary *pure-planned regime* by making it possible for enterprises to fulfill the *plan*, at the same time making other agents wealthier. Still, such a system is always far from being Pareto-optimal. Also, it is argued by Basu and Li (1998) that corruption plays a special role in transition economies, rather positive than negative, for it precedes economic growth (or even co-exists with it). But the authors were studying China, and we know that China is rather an exception among countries in transition, with respect to many parameters. As for standard transition economies, as well as for economies of the Third World, corruption has become a stable, persuasive and negative part of the public sector. For example, Wei (1997) conducts an empirical analysis of the impact corruption has on the propensity to invest; that is, how outside investors respond to corruption. Not only is the effect negative, but its magnitude is impressive. As a matter of fact, corruption suppresses outside investment, hence, economic growth as well.

On the whole, one can suggest that corruption could play a somewhat positive role in planned economies (like the former USSR), and to a certain degree in highly controlled transition economies (China), at the same time being an obstacle to economic prosperity in liberalized economies of a developing type such as those of the Third World and most of transition countries (e.g., today's Russia).

Things are even more complicated, however. It is insufficient simply to *allocate* a certain amount of resources to fighting corruption. One should rationally use these resources provided it is possible. Indeed, the history of corruption extends for thousands of years, and the attitude towards corruption was not always and everywhere as tolerant as in pre-revolutionary Russia (as for Russia, this is a distinct case, according to the classic literature, such as Gogol's Revisor, Ostrovsky's Dokhodnoe Mesto, and many others). At the same time, it would be incorrect to say that *entirely all* efforts to fighting corruption were bound to fail. Corruption is almost suppressed in many European countries; it is held within admissible limits in the USA and Singapore. However, historic conditions seem to play an important role and, for instance, the absence of corruption in Denmark and Singapore is due to quite different reasons (for example, in Singapure theft in the public sector is punishable by death). Therefore, the primary question is whether one could cope with corruption (or even suppress it entirely) using policy and punishment schemes not involving a dragon's measures,[1] in a country where corruption has become persuasive and stable enough.

## 1.2 Factors of corruption

A policy towards corruption should come from a deep understanding of its concrete form and characteristics, including the magnitude of losses resulting from corruption, mechanisms guaranteing its stability, and the environment which might be prolific for corrupt relations of the sort considered. Leaving aside the difficult and controversial question of losses from corruption, we will call *factors of corruption* everything that influences the level of corruption in a society, including stability of corruption equilibria, and so on.

Polterovich (1998) suggested the following classification of factors of corruption (this classification will be adopted, to some extent, in what follows).

[1]This is a famous Russian idiom meaning very cruel policy.

Factors of corruption are called *fundamental* if they arise from an imperfect environment; *organizational* if we mean a weak government; and *societal* if we refer to the strategic interaction between bureaucrats. One can go further and note that the first two types of factors are *external* in that they are considered as exogenous parameters to our system and have to do with the current economic situation, whereas factors of the third type could be called *internal*: they are determined endogenously in the system and usually make an *existing* equilibrium *stable*. We simply will call *fundamental* all the factors linked to the environment, and by *societal* we refer to the remaining ones. The terms *external* and *internal*, respectively, are equivalent to the two introduced above. As long as internal factors influence the interactions among bureaucrats, their appearance and work reveal *externalities* among the agents in the system under consideration.

Despite the huge variety of corrupt relations, almost every such kind is characterized by both external and internal factors. At the same time, there is a lack of models that take into account both types of factors in the current economic literature on corruption. Most of the existing papers appeal to factors either of external, or of internal nature (with the remarkable exception of Acemoglu and Verdier (1997)). This creates a number of problems, for such models usually do not reflect the complexity of real mechanisms underlying corruption in a given situation. As a result, one is bound to form an inadequate picture of what is really going on, which has undesirable consequences. On the basis of such models, it is dangerous to construct policy recommendations, as well as to simply make definite inferences.[2]

## 1.3 A fundamental factor

In the current paper the author intends to take into account external factors when determining the effect of internal ones, so to speak. To be more precise,

[2]Even when a given aspect of corrupt relations is under study, it is wise to take into account other aspects. As a matter of fact, as long as the corruption phenomenon is under study, a general idea of abstracting out from the real life picture in order to better illuminate features of current interest is not profitable, because factors of different nature, when determining the equilibrium level of corruption, interact more closely than someone would have suggested.

although the main subject of this work are externalities among bureaucrats (representing an internal factor of corruption) and their role in determining the equilibrium level of corruption, the model which is constructed for that matter is based on the production side of the economy. Namely, bureaucrats are assumed to take bribes from firms, hence, I study corruption in production process. I will analyse the impact of production functions on the corruption possibilities of bureaucrats, investigate mechanisms of deciding to become involved in corruption and choices regarding the amounts of bribes collected, and conduct a comparative statics relevant within the chosen approach. In addition, the production environment and relations will guarantee the existance of equilibria, which is crucial for justifying ongoing research.

To be more concrete about the production process (and its role in characterizing corruption possibilities as well), let us now describe a number of special cases of corruption in the production environment.

Producers consider corruption to be one of the most efficient strategies in a *rent-seeking war* where rent emerges from various artificial barriers. As for bureaucrats, corruption is an obvious and straightforward way to become richer. It is not a surprise, therefore, that illegal mutually profitable collusion called a corrupt act is a widespread phenomenon. Let us now characterize several types of such collusions.

**1. Tax evasion**

This variety of corruption cases should, from the government, or state, point of view, be rooted out. Indeed, after the state has chosen a certain tax scheme, it would be wise to implement it instead of allowing for some corruption in the tax collecting administration causing, as we know, additional distortions (see Shleifer and Vishny (1993)); the central government should better correct the tax scheme and maintain it as much as possible. That is why models of this kind of corruption should bear an enormous *normative* flavor by specifying conditions under which resources put into suppressing tax evasion will be allocated optimally, taking into account not only benefits but also costs of fighting such corruption. Presumably one of the most efficient measures is *provoking* corruption, that is, when a certain portion of bureaucrats (or, conversely, producers) are *artificial* in that they offer (or try to extort) bribes and then inform central authority which agents accepted this proposal. However, such a measure is sometimes considered to be *immoral* for it is associated with KGB practice, at least in Russia.

More scrupulous analysis should take into account the fact that tax evasion is a kind of a free rider's problem, since taxes are usually collected in order to provide various public goods (such as infrastructure, health and defence institutions and so on). Then, the primary question is how to *convince* tax payers that their money are used for good purposes. Unfortunately, in today's Russia, this is *not* the case, as it is commonly believed, and often taxes are simply lost, if not appropriated by influential authorities. Knowing that, it is hardly possible to refer to this social argument in collecting taxes. Anyway, this approach requires efforts in analysing the demand side of the economy, a consumers' income structure, gains and losses from corruption throughout the whole population, and so on. This line of research is quite popular now; see, e.g., Vasin and Panova (2000), Chander and Wilde (1992), Sanchez and Sobel (1993).

**2. Artificial cost reduction**

Examples of artificial cost reduction include the following: illegal access to a channel of a cheaper input, reduced provision of alternative resources used in production processes and allowance for the non-usage of anti-pollution technology, for instance, allowance to pollute the athmosphere, waters and ground with poison and radioactive waste. This branch is intimately connected with other branches in economic research, such as a general analysis of rent-seeking activities, a theory of externalities and so on. Probably, as far as pollution is concerned, corruption must be eradicated entirely, for should we refer to a cost-benefit analysis, trading off losses from corruption against expenses to fight it, we immediately find that one cannot assess losses adequately. Indeed, to do this, one must take into account future generations which probably will not favor today's policimakers for their comparative statics exercises. Even when the task to evaluate future ecological losses from corruption seems tractable, no one is able to choose a fair discount rate.

As for alternative inoffensive privileges, there is a question of how harmful corruption actually is. Indeed, regulation is a matter of distant government organizations (e.g., Parliament) and is unique and inelastic, so to speak, whereas in every specific situation knowledge of actual conditions may well turn out to be more important. Therefore, even from the government's position, suppressing corruption entirely may not be necessary, and the analysis of such controversial situations should presumably be of a positive nature, providing us with a deeper general comprehension of various mechanisms, such as how bureaucrats would respond to alternative anti-corruption measures and how a structure of corrupt relations (as well as the average level of corruption) would change if we try to gradually develop a new institutional design.

The next item is closely related to the one just described, but it contains quite interesting topics for discussion; besides, it lies at the base of the model introduced and studied below.

**3. Illegal provision of a complementary production input**

Under conditions of underdeveloped institutional infrastructure, which can be thought of as a common complementary resource, that is, a public good provision, it is often important for a producer to organize private, alternative resource, for his use. At this point, bureaucrats may be in a good position by possessing different such resources, e.g., having the discretion to dislocate firms elsewhere, or to open access to capacities of some state plant or factory, or to sell machinery or other objects which are formally written off. It would be naive to pretend to take into account all possible methods of illegal enrichment in exchange for a bribe. However, one could imagine a common remedy against corruption in all such situations: intensifying the provision of the public good in appropriate amounts. The neccesity of bribing officials would then seem to evaporate or be substantially reduced, together with the average corruption level. It is the formalization of the mechanisms described above which we call the impact of the fundamental, or external, factors on the main parameters of a model of a corrupt economy.

## 1.4 A societal factor

In contrast with the fundamental factor (which is the possibility of collusion between bureaucrat and producer due to an imperfect environment), societal factors do not allow for such explicit description. To begin with, let us recall that externalities occur whenever a utility of a given agent is directly affected by the other agents' actions. As everyone knows, it would be a mistake not to take external effects into account, for this results in a distorted view of a quantitative, and sometimes even qualitative, picture. An extreme case can take place when multiplicity of equilibria is being overlooked, or stability/instability of some equilibria is incorrectly prescribed, as a result of underestimating external effects.

Why is it logically natural to expect externalities to work when studying corruption? Imagine the following typical situation (see Polterovich (1998), and, within a somewhat more general framework, Sah (1991)). Assume that a fixed amount of resources, *a budget*, is allocated to fighting corruption. Provided violations of the law are rare enough, this budget is sufficient to fully screen all such cases, at least to make a punishment quite likely. On the contrary, if a situation is out of control, and law violations (corruption cases) abound, our budget is insufficient to adequately confront outlaws, since *any given violation* receives only a tiny fraction of resources, thus making corruption almost free of charge. As a result, the probability of detection is small.

Not only does a scenario where *burnt being caught* reveal these kind of externalities. A typical situation is when everybody knows what is going on and the costs from being corrupt take the form of *ill reputation*, or simply of a bribe to the superior bureaucrat. In both cases one can observe the same principle: when everyone is stealing, the reputation of being a thief is not that severe (everything is evaluated comparatively). In the latter scenario, the superior ultimately becomes rich enough by extorting bribes from many subordinates, which is probably reflected in charging smaller individual bribes. As a result, the suborninates' costs are reduced. This seems to be a suitable description of the highly corrupt authorities in the Russian economy.

From the formal point of view, we should assume that costs from corruption is a function depending not only on the level of individual involvement in corrupt relations by a given bureaucrat (shortly: an individual's corruption level), but also on every other bureaucrat's choice, thus being a function of many arguments (the same number of arguments as the number of bureaucrats). Naturally, costs are assumed to increase in the first argument (corresponding to the individual level of involvement in corruption) and decrease in the others (reflecting external effects just described). However, in order to simplify matters, as well as to follow the idea that bureaucrats are numerous and each of them have a negligible impact on the fundamentals of the economy, we will assume that the cost function depends only on two arguments, of which the first one represents an individual's corruption level, while the second one stands for the average bureaucrats' corruption level,[3] and that the costs are increasing in the first argument and decreasing in the second one.

[3]Polterovich (1998) simplifies the cost function (the "probability of detection" in his context) further on to depend *only* on the second argument, i.e., the average level, leaving aside the individual's choice. However, as we will see later, such a simplification results in the underevaluation of the role of external effects in determining the average corruption level.

If one is to follow literally the scenario just described, he/she at once confronts the exogenous nature of the external effects generated by bureaucrats among each other, in which case these externalities are just one of the characteristics of the economics of corruption. Therefore, one can do no better than to simply state its positive or negative role in determining the average corruption level. However, might we relax this implicit assumption of exogeneity of a *detection (and punishment) scheme* (see below), we immediately get a (possibly) strong instrument of economic policy: varying this scheme of punishment, one probably could essentially decrease the average corruption level *through the impact that the choice of a specific detection scheme has on the power of external effects among bureaucrats*, even remaining within the frame of the (*fixed*, for the time being) overall budget. And the primary problem with implementing such a policy measure consists of our undercomprehension of even the sign of its effect[4] on both the average and individual corruption levels. And, in turn, this sign depends on the sign of externalities' impact and on the direction in externalities' power evolution during the scheme's variation.

In this paper, the first step is made towards endogenizing punishment schemes and choosing the optimal one among all possible schemes or a certain sub-class of them. A special case is analysed where the cost function (which is an image, or a transformation, of a detection scheme) depends on two arguments: a personal degree of involvement in corrupt relations, and the overall (or the average) corruption level in a system under study. Of all such schemes, which one is the optimal, in the sense of minimizing the average corruption level? Put another way, how strongly should one refer to the average corruption level when considering a bureaucrat's corrupt actions? A formal conterpart of this question is not that simple since, while formalizing, one should have in mind the fixed budget assumption and take a formal account of it in a precise manner (or, at least, implicitly). Otherwise a simple increase in the amount of resources to fight corruption definitely suppresses

[4]This contrasts with some popular measures against corruption, such as increasing penalties or enlarging the budget allocated to suppressing corruption etc., when one can be confident of the sign of the effect of their implementation. Unfortunately, the latter ones often require substantial resources when being implemented. The variation of the detection scheme *does not* require that many resources, but, vice-versa, one should check that he/she is not doing a bad thing for society by altering the scheme of punishment.

the average level.

For more on these issues, see Chapter 3. In the next chapter the basic model of corruption in production process is introduced and analysed, to a certain degree. Then, I use this model to compare alternative punishment schemes, which is presented later in the paper.

# 2 A basic model of corrupt relations in the production process

## 2.1 Model specification

We start with an economy consisting of producers and officials interacting with each other in the following manner. Every official is associated with a certain group of producers, and can serve the group in some illegal manner, in exchange for bribes. Every official chooses the scale of an illegal activity (i.e., the amount of service provided for a given producer) to be an arbitrary nonnegative real number. The only thing that worries him is the possibility (and the degree) of punishment which increases with the level of aggregated illegal service supplied by this official to all his or her producers. (Sometimes we refer to the level of aggregate illegal service produced by a given official as *the degree of personal involvement in corruption*, in contrast with the overall corruption level, which is the average of all the individual degrees.) Generalising, we call this service an *illegal resource supply.* However, costs from being corrupt are partially mitigated by externalities emerging among bureaucrats when they make their corrupt deals. Specifically, the higher the average level of involvement in corruption (shortly, the corruption level), the lower are the costs shared by every given bureaucrat.[5] Below is the mathematical formalization of this mechanism.

Assume $i$–th bureaucrat is serving $N_i$ producers and supplies $j$–th producer with an $q_{ij}$–amount service, where $q_{ij} \in [0, 1]$ or $[0, +\infty)$. We focus our analysis on the last case for the time being. The scale of aggregated illegal activity of a given bureaucrat (we will also refer to it as the *degree of a personal involvement*) equals $q_i = \sum_j q_{ij}$. Let us state that the costs from corruption are $\varphi(q_i, q_{\text{av}})$, where $q_{\text{av}}$ is the average corruption level. The cost

[5] Recall that this is a somewhat simplified version of a general problem where costs are determined by the profile of all bureaucrats' corruption levels.

function is assumed to satisfy the following conditions:[6]

$$\begin{gathered}\varphi(0, q_{\mathrm{av}}) = 0, \quad \frac{\partial \varphi}{\partial q_i}(q_i, q_{\mathrm{av}}) > 0, \quad \frac{\partial \varphi}{\partial q_{\mathrm{av}}}(q_i, q_{\mathrm{av}}) < 0, \\ \frac{\partial^2 \varphi}{\partial q_i \partial q_{\mathrm{av}}}(q_i, q_{\mathrm{av}}) < 0, \quad \frac{\partial^2 \varphi}{\partial q_i^2}(q_i, q_{\mathrm{av}}) > 0, \\ \varphi(q, q) \quad \text{is a convex function of} \quad q. \end{gathered} \tag{1}$$

The first assumption states that irrespective of the average level, an honest agent bears no costs. The second and the third ones express in a formal way that costs from being corrupt increase with the personal degree of involvement and decrease with the second argument representing the average corruption level in the system. The forth assumption states that marginal, not only total, costs decrease when the average corruption level rises (this is quite natural: an increase in the average degree of involvement in corruption changes the overall *scale of measurement* of corruption, thus, its measurement units as well). The fifth one is a standard convexity assumption.

The only controversial assumption is the sixth one. Let us justify it by investigating the nature of the external effects reflected in the negative dependance of the cost function on its second argument. Imagine a society comprised of identical bureaucrats perfectly repeating each others' actions (hence, their choices coincide). Naturally, in such a society, each agent's costs are equal to

$$\psi(q_i) = \varphi(q_i, q_i). \tag{2}$$

The sixth assumption reflects the fact that convexity still holds for this imaginary society: otherwise simultaneous increase of all corruption levels produces economies of scale in costs, which would inevitably lead to a "corruption bubble;" we rule out this possibility here.

Now we introduce a basic notion. We say that the external effects are *absent* if the cost function of any given agent depends only on his own corruption level and precisely according to (2), i.e., *as if he were placed into a society of identical agents perfectly repeating his actions.* One could think of this definition in the following manner: a society is heterogeneous, and every type of agent is represented by many of them, and the central authority balances the budget in a way that guarantees independence of different groups'

[6]Note that $\varphi$ could exhibit a jump in zero rising from, say, moral costs following dishonest behavior or from the possibility of revealing a corrupt act per se. When analysing the situation, one must carefully distinguish a case with a jump.

choices. As for the possibility of maintaining the overall budget fixed, this is arguable; still, fluctuations of the budget required are believed to be insufficient *provided groups are numerous and truly independent in their actions.*

However, in the spirit of my research, *it is* interactions between groups that matter; hence, the absence of externalities is imaginary. We refer to it only as a *reference point*, as will be explained later on when turn to the comparison of alternative punishment schemes.[7]

I have characterised costs from corrupt activity. As for the benefits it promises officials, they result from bribe collections. Every given bureaucrat extort bribes in exchange for the illegal services he offers to producers under his jurisdiction. In the current setting (throughout the paper) we are not taking into account the fact that, ceteris paribus, the higher the bribe extorted, the more probable is its detection (such a relaxation corresponds to the case where the mechanism of detection does not include *denunciations* at all, being directly linked to an illegal activity per se).

Recalling that an official is a monopolist in providing illegal services to his producers, we will assume that he extorts the maximum that every producer is willing to pay for this service. Such an assumption is extreme in (at least) two aspects. First, it implies common knowledge of the production possibilities, which is a good approximation only when *repeated interactions* are under study. But repeated interactions give rise to various aspects (such as dynamics) not touched upon here. When assymmetric information is present, the more realistic is the assumption that an official charges a unique price for his/her service, as in Polterovich (1998). Secondly, even when bureaucrats are well informed about technology, some bargaining power is in producers' hands and, e.g., a Nash bargaining solution could be a better approximation. However, we ignore here these objections, assuming extortion of the maximum feasible bribe.[8] In order to characterise this maximum bribe, we now

[7]Probably one suggests that we can model a free-of-externalities scheme in an alternative manner, namely as $\psi(q_i) = \varphi(q_i, 0)$. However, following our general methodology, this scheme corresponds, on the contrary, to a situation of individual deviation from honest behavior, when all the other agents are not involved in corrupt relations. The formula given above shows costs from such a deviation and plays a crucial role in determining whether a corruption-free equilibrium actually exists.

[8]A case of a Nash bargaining solution seems to deviate not very far from that considered, until comparative statics with respect to the relative bargaining power of bureaucrats and

proceed to a description of the production process.

In the $i$-th region, the $j$-th producer possesses a production function of the form $F_{ij}(M_{ij}, g_i, q_{ij})$ which depends on three arguments: the money stock, the level of public good provision, or *infrastructure*, in this region,[9] and illegal services supplied by the i-th region's official.[10] The production function is assumed to satisfy the following (more or less standard) conditions (omitting indices of region and of producer, as well as arguments of the production function; instead, subscripts denote marginal factors, from now on):

$$\begin{array}{cc} F(0,0,0)=0; \qquad F'_i > 0, \quad i=1,2,3; \\ F''_{ii} < 0, \quad i=1,2,3; \qquad F''_{1i} > 0, \quad i=2,3. \end{array} \tag{3}$$

The first three properties show that all the factors are essential in production, but the production function exhibits diminishing returns, with respect to them. The last one means that the *marginal* productivity of the money factor increases with the rise in either the provision of a public good or an illegal service. As for the sign of the last mixed derivative, $F''_{23}$, it is not specified. We will find in 2.2 that this sign crucially affects the efficiency of a given anti-corruption measure, namely, an additional provision of a public good.

In the case of the refusal to provide an illegal service (or to accept this service), a producer has $F_{ij}(M_{ij}, g_i, 0) \neq 0$[11]; otherwise, if an illegal act occurs and a bribe of $b_{ij}$ is paid, a producer has $F_{ij}(M_{ij} - b_{ij}, g_i, q_{ij})$. According to the approach chosen, we should use the following equation to determine the amount of an individual bribe:

$$F_{ij}(M_{ij} - b_{ij}(q_{ij}), g_i, q_{ij}) = F_{ij}(M_{ij}, g_i, 0). \tag{4}$$

producers is conducted.

[9]It is possible that a public good is provided in a centralized manner, i.e., uniformly across all the regions in our system. Although we omit such a case here, comparing it to the one considered in the paper may turn out to be beneficial for our complete understanding of corruption in production processes.

[10]One could introduce *several* arguments of any of these three different types, but probably it will generate no new ideas. It seems sufficient to refer to these basic arguments as *generalised* ones.

[11]I assume that the price of a product is constant and unique.

The overall bribe collection of a given bureaucrat then equals $\sum_j b_{ij}(q_{ij})$, provided our bureaucrat has come into collusion with every producer under his jurisdiction, dividing the overall illegal services of $q_i$ between them in a certain manner. Note that the overall bribe collection does not exceed $\sum_j M_{ij}$, which is the overall money stock of the i-th region's producers.

Of course, whatever $q_i$ has been chosen, an official will try to allocate it optimally between firms. Hence, every bureaucrat first solves the following problem:

$$\begin{array}{c} \sum_j b_{ij}(q_{ij}) \longrightarrow \max, \\ s.t. \quad \sum_j q_{ij} \leq q_i, \quad q_{ij} \geq 0, \end{array} \tag{5}$$

for all $q_i > 0$. We begin with the analysis of this very problem.

First of all, it is necessary to study the behavior of functions $b_{ij}$ which are known to us as implicit solutions of (4). The rule of implicit functions' derivation leads us to the following equation (subscripts are omitted again):

$$-F_1'(M - b(q), g, q) \cdot b'(q) + F_3'(M - b(q), g, q) \equiv 0; \tag{6}$$

thus, increasing q, one increases the value of the bribe necessary to be provided with $q$ units of an illegal resource (as expected).

Taking the second derivatives (and omitting arguments also), we get:

$$F_1' \cdot b''(q) = F_{11}'' \cdot b'^2(q) - 2F_{13}'' \cdot b'(q) + F_{33}''. \tag{7}$$

We infer, therefore, that functions $b_{ij}(q_{ij})$ are concave, as the corollary of the properties of production functions claimed in (3). Also, it is not difficult to calculate the derivative of $b_{ij}$ at zero, using (6): it equals $\dfrac{F_3'(M, g, 0)}{F_1'(M, g, 0)}$.

Let us now summarize the properties of functions $b_{ij}(q_{ij})$ (omitting subscripts of the corresponding production function):

$$\begin{array}{c} b_{ij}(0) = 0, \qquad b_{ij}'(q_{ij}) > 0, \qquad b_{ij}''(q_{ij}) < 0, \\ b_{ij}'(0) = \dfrac{F_3'(M, g, 0)}{F_1'(M, g, 0)}, \qquad b_{ij}(q_{ij}) \leq M. \end{array} \tag{8}$$

Starting from these properties, one could formulate and prove the following assertion.

**Theorem 1** *For every $q_i$, problem (5) has a unique solution. Denote the resulting amount of the bribe collection by $b_i(q_i)$. Then, the following properties* *hold:*

$$\begin{aligned} b_i(0) = 0, \quad b_i'(q_i) > 0, \quad b_i'' < 0, \\ b_i'(0) = \max_j(b_{ij}'(0)), \quad b_i(q_i) \textit{ is bounded above.} \end{aligned} \tag{9}$$

For **Proof**, see Appendix.

We now introduce an equilibrium concept to our economy. Let us start with the agent's problem. Provided there are many agents in our system, every given agent's choice of his/her level of involvement in corrupt relations has an infinitesimal impact on the average level of corruption. Neglecting this impact, we wrote the agent's problem like

$$b_i(q_i) - \varphi(q_i, q_{\mathrm{av}}) \longrightarrow \max_{q_i \geq 0}. \tag{10}$$

**Definition.** We call *corruption equilibrium* a bundle $(q_{\mathrm{av}}; q_1, q_2, \ldots.., q_K)$, where $K$ is the number of agents (i.e., bureaucrats), such that, for any $i$, $q_i$ is a solution to (10), and, in addition, a *balance condition* holds which requires that

$$q_{\mathrm{av}} = \frac{1}{K} \sum_i q_i. \tag{11}$$

Before turning to the detailed equilibrium analysis including agents' behavior and comparative statics with respect to various parameters, we will consider a digression concerning the role of a public good in determining the equilibrium level of corruption.

## 2.2 The role of public good provision

Consider the economy described above. Assume that the level of public good provision is an exogenuous variable in our system thus serving as an instrument at the hand of the government. What consequences are brought about by an increase/decrease in this exogenuous parameter?

In such a situation, we are primarily interested in the effect that a change in public good provision has on functions $b_{ij}(q_{ij})$. Omitting subscripts, we should analyse the behavior of an implicit function satisfying the following equation:

$$F(M - b(g, q), g, q) = F(M, g, q), \tag{12}$$

when $g$ changes. Taking the derivative of (12) with respect to $g$, one gets:

$$\frac{\partial b}{\partial g} = \frac{F_2'(M-b,g,q) - F_2'(M,g,0)}{F_1'(M-b,g,q)}. \tag{13}$$

The denominator is positive while the sign of the numerator depends crucially on the sign of a mixed derivative, $F_{23}''$ (recall that we have not yet specified this sign). Provided a public good and an illegal resource are substitutes, one has $F_{23}'' < 0$; hence (using properties of production functions postulated in (3),

$$F_2'(M-b,g,q) < F_2'(M,g,q) \leq F_2'(M,g,0). \tag{14}$$

This means that the maximum bribe decreases when $q$ rises. Moreover, the same holds if a public good and an illegal resource are *weak complements*. Thus, in the two cases just described, corruption opportunities of bureaucrats become worse off with the increase in public good provision. It is easy to show that, in turn, this leads to a decrease in the average corruption level.

On the contrary, if corruption deals result in an improved returns on the public good factor, the effect could be of the opposite sign (though being slightly mitigated by a decreased money factor: a part of a producer's money is spent on the corresponding bribe). Therefore, when considering such a measure (namely, increasing public good provision), one should analyse the specific features of the production process.

Consider an example. Let the production function be

$$F(M,g,q) = M^{\alpha}(g+q)^{\beta}, \quad \alpha, \beta > 0. \tag{15}$$

One can see that instead of a poorly provided public good, a bureaucrat supplies producers with an absolutely identical factor of production, but illegally and in the private form (for example: local channels on TV, or simply doing his obligations but in exchange for bribes). Then, a mixed derivative which is of our interest is

$$F_{23}'' = M^{\alpha}\beta(\beta-1)(g+q)^{\beta-2}, \tag{16}$$

and it is negative only for $\beta < 1$. Hence, a public good and an illegal resource are substitutes for $\beta < 1$, and complements for $\beta > 1$. However, the necessary bribe, in fact, decreases for all $\beta$. To prove this, we just solve (15) explicitly. What we get is the following formula for the bribe:

$$b(g,q) = M \cdot \left[1 - \left(\frac{g}{g+q}\right)^{\beta/\alpha}\right] = M \cdot \left[1 - \left(1 - \frac{q}{g+q}\right)^{\beta/\alpha}\right]; \tag{17}$$

obviously, bribe $b$ is a decreasing function of $g$.

## 2.3 Equilibrium analysis

We have introduced the notion of an equilibrium in section 2.1. Now we examine its existence, uniqueness and stability properties. In order to avoid excessive subscripts and tedious calculations, we reorganize our basic model a little bit. Assume that there is a *community* of bureaucrats each of them characterized by his *type*, $\theta$, and a distribution function $F(\theta)$ is given explicitly. The type of an agent represents his corruption opportunities. More precisely, assume that a common *corruption opportunity function* $b(q, \theta)$ is given, which depends on the two arguments and is characterized, for a fixed $\theta$, by the properties postulated in (9), at the same time being an increasing function in $\theta$, together with its derivative with respect to $q$. Thus, the higher is $\theta$, the better are the corruption opportunities of an agent of type $\theta$. Below we summarize the properties of function $b(q, \theta)$:

$$\begin{gathered} b(0,\theta)=0, \qquad \frac{\partial b}{\partial q}(q,\theta)>0, \qquad \frac{\partial^2 b}{\partial q^2}(q,\theta)<0, \\ \frac{\partial b}{\partial \theta}(q,\theta)>0, \qquad \frac{\partial^2 b}{\partial q \partial \theta}(q,\theta)>0, \\ \forall\theta \quad \exists \quad \hat{q}(\theta): \qquad \frac{\partial b}{\partial q}(\hat{q}(\theta),\theta)=0. \end{gathered} \tag{18}$$

The last condition slightly differs from the assumption that the function $b(q, \theta)$ is bounded above, which is a direct consequense of (9). However, the generality of this discussion is not strongly limited: in the case when $q$ varies on $[0, +\infty)$ one can simply implement a corresponding monotonic reparametrization with respect to $q$.

Besides, for the purpose of convenience, we also make a monotonic transformation with respect to $\theta$, such that

$$\theta \equiv \hat{q}(\theta). \tag{19}$$

This is possible since $\hat{q}(\theta)$ is strictly increasing in $\theta$, which follows directly from the postulated properties (18).

The problem of an agent of type $\theta$ looks as before in (10):[12]

$$b(q,\theta) - \varphi(q, q_{\text{av}}) \longrightarrow \max_{q \geq 0}, \tag{20}$$

[12] Again, we assume no individual agent affects $q_{\text{av}}$ when choosing his $q(\theta)$. From the

where $\varphi(q, q_{\text{av}})$ is the function characterizing costs of corruption (see subsection 2.1). Here we reproduce its properties again:

$$\begin{gathered}
\varphi(0, q_{\text{av}}) = 0, \qquad \frac{\partial \varphi}{\partial q}(q, q_{\text{av}}) > 0, \qquad \frac{\partial \varphi}{\partial q_{\text{av}}}(q, q_{\text{av}}) < 0, \\
\frac{\partial^2 \varphi}{\partial q \partial q_{\text{av}}}(q, q_{\text{av}}) < 0, \qquad \frac{\partial^2 \varphi}{\partial q^2}(q, q_{\text{av}}) > 0, \\
\varphi(q, q) \quad \text{is a convex function.}
\end{gathered} \tag{21}$$

The first-order conditions for (20) take the form of

$$\begin{gathered}
\frac{\partial b}{\partial q}(q, \theta) \leq \frac{\partial \varphi}{\partial q}(q, q_{\text{av}}), \\
\text{with strict equality for} \quad q > 0.
\end{gathered} \tag{22}$$

**Definition.** A bundle $(q^*_{\text{av}}, q^*(\theta))$ is said to be an equilibrium if, for every $\theta$, a value $q^*(\theta)$ solves problem (20) when $q_{\text{av}} = q^*_{\text{av}}$, and the balance condition holds:[13]

$$\int q^*(\theta) dF(\theta) = q^*_{\text{av}}. \tag{23}$$

One can check that the model with a finite number of bureaucrats introduced in section 2.1 is in fact a special case of the one just described, and corresponds to a discrete distribution function $F$. In the general case, we assume that the expected value of the type is finite:

$$\theta_{\text{av}} = E\theta = \int \theta dF(\theta) < +\infty. \tag{24}$$

We now turn to the analysis of equilibria in this economy, proceeding in three steps. First, we study the behavior of agents when $q_{\text{av}}$ is fixed. In the next step, we aggregate individual choices into a *reaction function*:

$$\Psi(q_{\text{av}}) = \int q(\theta, q_{\text{av}}) dF(\theta), \tag{25}$$

mathematical point of view, this means that the distribution $F$ has no atoms, or, at least, all atoms have a negligible size.

[13] Note that this modification of the basic model admits another interpretation: a game of incomplete information! Agents know their own types and, in addition, a precise form of the distribution function over the set of all types. Based on these data, every agent chooses his $q$. The definition introduced above is exactly a definition of a Bayesian–Nash Equilibrium for this game.

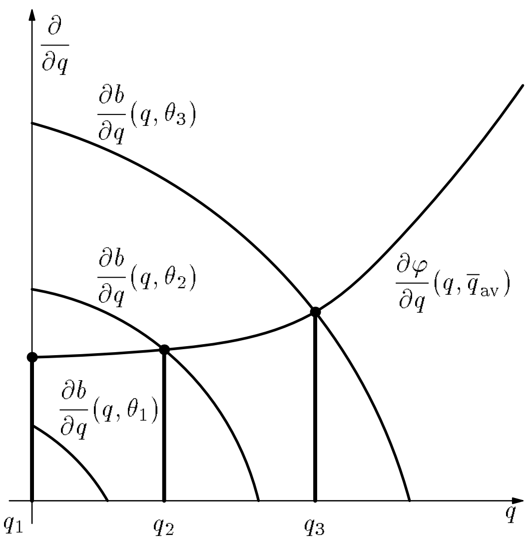


Figure 1: First-order conditions ($\theta_1 < \theta_2 < \theta_3$)

where $q(\theta, q_{\mathrm{av}})$ is the choice of an agent of a type $\theta$, provided the average level of corruption equals $q_{\mathrm{av}}$. One can easily note that equilibria are then represented by points of intersection of the reaction function with the diagonal line; in the final step, we prove the existence and study some properties of equilibria in our economy.

As a first step, assume that $q_{\mathrm{av}}$ is fixed. First order conditions (22) have, depending on $\theta$, an interior (positive) or a corner (zero) solution. Check that the right hand sides of (22) are identical for all $\theta$. Let us illustrate this situation graphically (see Figure 1). Note, however, that the first-order conditions correctly specify agents' choices only in the continuous case. If, on the contrary, the cost function exhibits a jump in zero (denote the jump value by $\varphi_0$), than part of the agents for whom the resulting utility (i.e., the utility when choosing the point of intersection) is less than zero would prefer to abstain from corruption.

For the time being, we denote the *actual* choice function by $\bar{q}(\theta, q_{\mathrm{av}})$, retaining our old notation $q(\theta, q_{\mathrm{av}})$ for the solution of the equation (22). We also continue to use the term *resulting utility* for the utility of an agent who has chosen $q(\theta, q_{\mathrm{av}})$. Thus, the resulting utility could be either positive or negative while the *actual* utility is always a nonnegative number.

**Lemma 2.1** *The resulting utility is a nondecreasing function of $\theta$. As a consequence, provided there is a jump in zero, the agents' choice function has the form*

$$\bar{q}(\theta, q_{\mathrm{av}}) = \begin{cases} q(\theta, q_{\mathrm{av}}), & \text{for } \theta \geq \bar{\theta} \\ 0, & \text{for } \theta < \bar{\theta} \end{cases}, \tag{26}$$

*where $\bar{\theta}$ satisfies the following equation:*

$$b(q, \bar{\theta}) - \varphi(q, q_{\mathrm{av}}) - \varphi_0 = 0. \tag{27}$$

**Proof.** One can either take the derivative of $b(q, \theta) - \varphi(q, q_{\mathrm{av}}) - \varphi_0$ in a straightforward way, having (22) in mind, or refer to the Envelope Theorem (see, e.g. Mas-Colell, Whinston, Green (1995), p. 964) which states that a full derivative of a maximand (here of the resulting utility function) with respect to some parameter equals the partial derivative with respect to this parameter taken in the argmaximum point. In our case, this partial derivative is $\partial b/\partial\theta$, and the assertion follows from properties (18). Particulary, there exists at most one cut-off level $\bar{\theta}$ for which the resulting utility equals zero. Agents whose type $\theta < \bar{\theta}$ would prefer not to be involved in corruption at all, others will choose $q(\theta, q_{\mathrm{av}})$. The proof is complete.

**Lemma 2.2** *The agents' choice function is nondecreasing in $\theta$, continuous for continuous cost functions, and discontinuous for cost functions exhibiting a jump in zero. The actual utility function is always continuous and increasing.*

**Proof.** We simply take the derivative of the solution to (22) with respect to parameter $\theta$, using the properties of the corruption opportunities function:

$$\frac{\partial q}{\partial \theta} = -\frac{\partial^2 b/\partial q \partial \theta}{\partial^2 b/\partial q^2} > 0. \tag{28}$$

Hence, the choice function $q(\theta, q_{\mathrm{av}})$ is continuous and increasing in $\theta$. The actual choice function, $\bar{q}(\theta, q_{\mathrm{av}})$, completely coincides with $q(\theta, q_{\mathrm{av}})$ when the cost function is continuous, and coincides with it *starting from* $\bar{\theta}$ when there

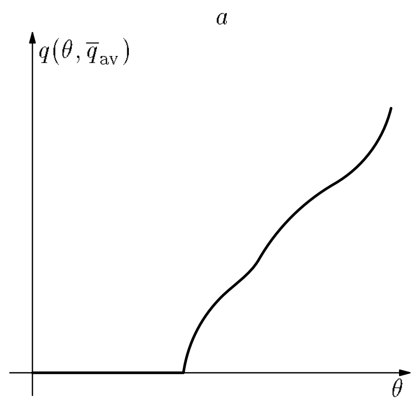


Figure 2: Agents choice function, $q(\theta, \bar{q}_{\rm av})$ (the continuity case)

is a jump in zero. Properties of the actual utility function are direct consequences of the properties of the resulting utility. The proof is complete.

Figures 2 and 3 illustrate agents' choice functions, depending on whether the cost function exhibits a jump in zero.

The next step is to vary $q_{\rm av}$ and analyse the reaction function $\Psi(q_{\rm av})$. The behavior of individual choices when $q_{\rm av}$ varies is characterized in the next lemma.

**Lemma 2.3** *When $q_{\rm av}$ increases, an agent's choice function cannot decrease; the same holds for the agent's utility.*

**Proof.** This time, all that changes is the *right hand side* of the first-order conditions (22). For an interior solution, we have

$$\frac{\partial q}{\partial q_{\rm av}} = \frac{\partial^2 \varphi / \partial q \partial q_{\rm av}}{\partial^2 b / \partial q^2} > 0, \tag{29}$$

using (18), (21). Repeating this exercise for an agent's utility, one should just recall the Envelope Theorem (a partial derivative of the utility function with respect to $q_{\rm av}$ equals $-\dfrac{\partial \varphi}{\partial q_{\rm av}} > 0$). If the corner solution takes place, we have $\bar{q} = 0$, and the actual utility equals zero as well, hence, both could only increase or remain the same. The proof is complete.

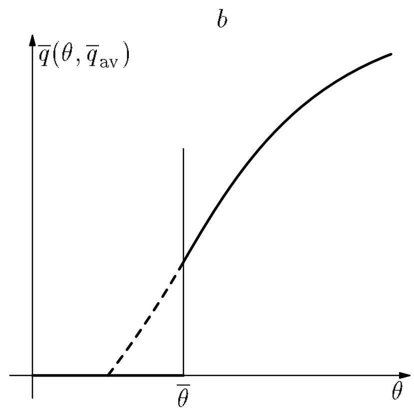


Figure 3: Agents choice function, $\bar{q}(\theta, \bar{q}_{\text{av}})$ (the case of a jump in zero)

We illustrate this fact graphically (see Figure 4). When $q_{\text{av}}$ increases, the curve representing marginal costs shifts downwards. The family of marginal revenue curves remains the same. Hence, the choice of a given agent cannot decrease (some agents, as one can see from the figure, may even become corrupt due to such a change).

**Lemma 2.4** *The reaction function $\Psi(q_{\text{av}})$ is nondecreasing and bounded above.*

**Proof.** The nondecreasing property of $\Psi$ follows from its integral representation (see definition (25)) and from Lemma 2.3. Moving further along, one can note that the last property in (18) guarantees that the choice of an agent with type $\theta$ could not exceed $\hat{q}(\theta)$. Recalling our reparametrization (19) and, finally, the finiteness assumption (24), we can construct a chain of inequalities:

$$\Psi(q_{\text{av}}) = \int q(\theta, q_{\text{av}}) dF(\theta) \leq \int \theta dF(\theta) = \theta_{\text{av}} < +\infty, \tag{30}$$

uniformly with respect to $q_{\text{av}}$. The proof is complete.

Now we proceed to the final step of the analysis. One can observe that stable equilibria are those rest points of a $\Psi$-mapping for which the reaction curve intersects the diagonal line from above (see Figure 5).[14] But such points

[14] Indeed, a standard iterating approach argument is applicable in our situation (see, e.g. Mas-Colell, Whinston and Green (1995), Chapter 17, p. 620).

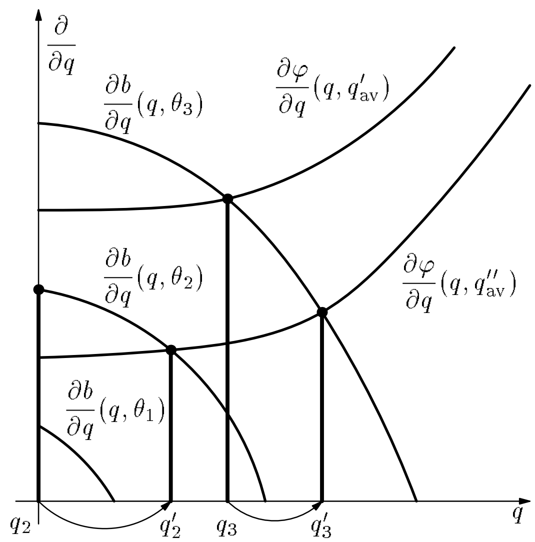


Figure 4: The effect of increasing $q_{\text{av}}$ ($q'_{\text{av}} < q''_{\text{av}}, \quad \theta_1 < \theta_2 < \theta_3$)

always exist, as follows from the Tarskii's Fixed Point Theorem reproduced below.

**Theorem 2** *A nondecreasing and bounded mapping of the closed interval* $[0, +\infty)$ *into itself has at least one stable rest point.*

Note that continuity is not required here. We are not going to prove this theorem, for it is a pure mathematical fact. However, let us discuss it. Look at Figure 6. Either one has a $\Psi$-curve which lies entirely below the diagonal line (in which case the only equilibrium is zero, and it is stable), or there is a nonempty subset of positive real numbers such that, the $\Psi$-curve lies above the diagonal line in corresponding points. Taking the suppremum of this set, one gets the stable equilibrium (in fact, the one with the maximum level of corruption).

Let us summarize our findings.

**Theorem 3** *The model characterized by (20) and (23) always admits at least one stable equilibrium. In every stable equilibrium, a choice of an agent is*

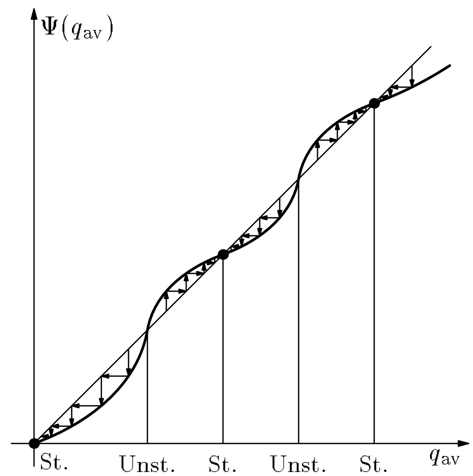


Figure 5: Stable and unstable equilibria

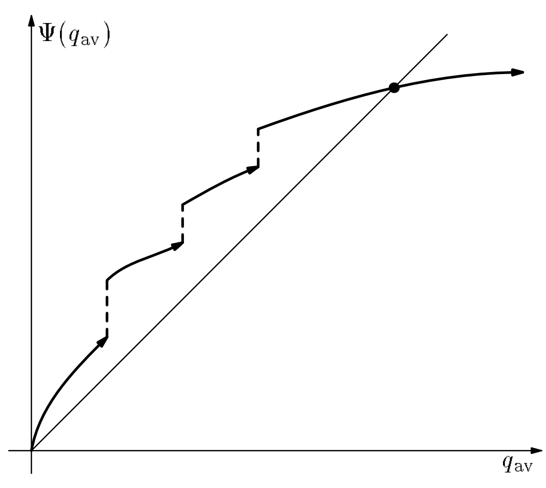


Figure 6: The Tarskii's Fixed Point Theorem

*a nondecreasing function of $\theta$. If the cost function has a jump in zero, then there exists a cut-off value $\bar{\theta}$ such that for $\theta < \bar{\theta}$, choices of agents $\bar{q}^*(\theta) = 0$, while $\bar{q}^*(\bar{\theta}) > 0$. In the case of continuity of the cost function, the choice function $q^*(\theta)$ is continuous. The utility function is always continuous and nondecreasing, with respect to $\theta$.*

So, we have established the existence of at least one stable equilibrium. Usually (especially when the cost function exhibits a jump in zero) the model described above allows for *two* stable equilibria, of which the one is corruption-free and the other is characterized by a high level of corruption. Also, there exists an unstable equilibrium which serves as a boundary point between zones of attraction. All in all, however, the precise number of equilibria depends on some additional assumptions, and it is not easy to characterize all the possible situations. The question of stability of a given equilibrium is quite difficult as well, even when the corruption-free equilibrium is concerned. We left these details alone in the current paper, assuming instead that our system "lives" in one of the stable *corruption* equilibria, and study comparative statics around it.

The first question one probably wishes to ask regards that of a comparison between long-run and short-run perspectives when considering a change in a certain parameter. In the framework of the static model, the possibility to discuss short- and long-run periods may seem amazing; still, it sometimes exists. We simply take that in the short-term period a new $q_{\rm av}$ is being determined through the choices of agents (after a change in parameters occurs) holding their expectations fixed (hereby, assuming the agents' best responses on the old $q_{\rm av}$). But in this case it is nothing else than the definition of the reaction function! Thus, we take the short-run reaction to the parameters' change to be equal to

$$\tilde{q}_{\rm av} = \Psi(q_{\rm av}, \gamma), \tag{31}$$

where $\gamma$ is the parameter (or a group of parameters) under consideration. As for the long-run, it is a new equilibrium, i.e., the solution to the equation[15]

$$q_{\rm av}(\gamma) = \Psi(q_{\rm av}(\gamma), \gamma). \tag{32}$$

[15]We require that, when possible, after a small change in parameters, the system doesn't switch to a *qualitatively* new equilibrium. Thus, we are searching for the solution to (32) close to the previous one.

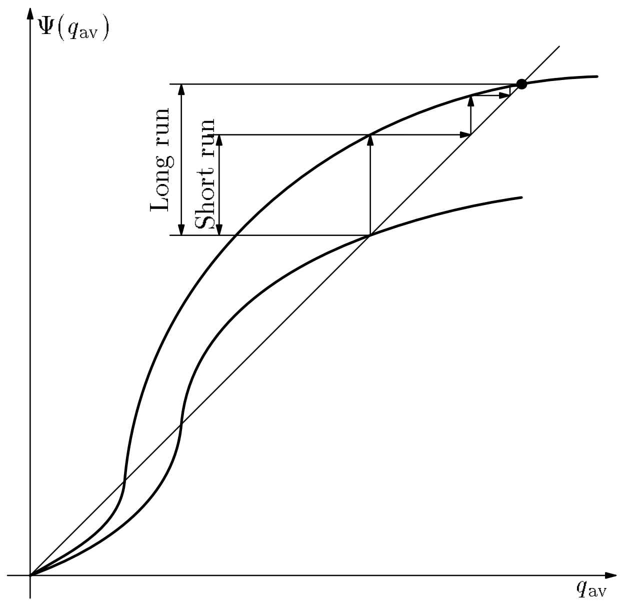


Figure 7: Comparative statics: short- and long-run perspectives

A typical situation is illustrated by Figure 7. Suppose some parameters of our system change. Provided it used to be in a stable equilibrium, the system (i.e., the average corruption level) moves from $q_{\mathrm{av}}$ to $\Psi(q_{\mathrm{av}})$ in the short-run. However, in the long-run, our system approaches a new equilibrium, and the peculiarity of the model considered is that the long-run effect *exeeds* the short-run one in absolute terms. This is easily seen from Figure 7: the nature of this phenomenon is the same as that of a multiplicity of equilibria, namely, it comes from an increasing property of $\Psi$ (we omit the technical details of this reasoning). Thus, we have proved the following important theorem.

**Theorem 4** *In the model considered, long-run shifts in the average level of corruption, in response to alternative parameters' changes, exceed those in the short-run perspective.*

Concluding this chapter, recall that the model introduced above has applications going far beyond the scope of corrupt relations in the production process; moreover, and beyond the scope of corruption in general, having to deal with a broad class of *crime and punishment* situations. One can check that the only crucial assumption for its applicability is that this kind of criminal activity be of Becker's type, i.e., where individuals compare costs (the risk of detection) and benefits (usually monetary) from crime (and, in addition, that agents make continuous choices). Then, most of the properties of functions $b(q,\theta)$, $\varphi(q,q_{\mathrm{av}})$ still hold, together with the mathematical statement of a problem per se. I use here the term "corruption" since historically the most important questions arose in the context of corruption, but our main conclusions are relevant for theft, tax evasion, etc. Probably corrupt relations are characterized by a huge differentiation in opportunities (hence, in possible benefits), which is a justification of introducing explicitly various types of agents. However, the basic question raised here is important in all the scenarios listed above (and in many other ones, as well). Namely: how do variations in the scheme of punishment affect the average corruption (or a criminal) level?

# 3 The role of external effects among bureaucrats

## 3.1 Alternative punishment schemes

From the formal point of view, a scheme of punishment is characterized by the consequences of its implementation, that is, by a function showing costs from corrupt activity which are shared by bureaucrats. The previous chapter dealt with a fixed cost function linked to a definite (implicit) scheme of punishment. In the current chapter, on the contrary, we state the problem of choosing the *optimal* (at least, second-best) scheme, thus considering (and comparing) different schemes simultaneously. This takes the form of choosing the optimal cost function $\varphi(q, q_{\rm av})$,[16] according to a definite criterion. Throughout the paper, our criterion would be to minimize the average corruption level, which, in turn, is the solution to problem (20) and (23) studied in the previous chapter.

Apparently our task is trivial unless we introduced a sort of a budget constraint: one simply should specify $\varphi$ to be infinitely large for all pairs of arguments. In order to make sense, we must take into account a fixed budget which the policymaker could operate. A straightforward way here would be to introduce a budget constraint explicitly, but it is not that simple for reasons of strategic interactions between the policymaker and bureaucrats: the budget constraint itself depends on the decisions of the bureaucrats. Hence, this is a new story, and it is left for further research. In the current context, I elaborate on some trivial consequences of a fixed budget, in the framework of the model studied. Imagine (as in the previous chapter) a society consisting of identical bureaucrats all of them choosing one and the same level of corruption equal to $q_{\rm av}$. The cost burden of any given bureaucrat *does not depend on the scheme of punishment being chosen* and is determined exclusively by the overall amount of resources allocated for fighting corruption.[17] In a formal

[16] More generally, one wishes to choose a function $\varphi(q(\cdot))$ depending on the whole profile of individual corruption levels, though in the current context this problem is a bit simplified.

[17] It is true, unless bureaucrats could be distinguished from each other, having a sort of *label*, and this labelling is *deliberately* taken into account. We exclude this possibility from

way (taking into account a fixed budget implicit condition), let us state that function

$$\psi(q_{\rm av}) = \varphi(q_{\rm av}, q_{\rm av}) \equiv const \tag{33}$$

is an exogenuos characteristic of our system (for *this* is the cost function for identical bureaucrats in a society where $q_{\rm av}$ is the average level of corruption (hence, $q_{\rm av}$ equals individual levels as well). We call this introduced one–argument function a *diagonal* of the function $\varphi$, due to the nature of its construction. Thus, our (last) binding assumption is that the policymaker could not affect the diagonal of the cost function.

Even when this requirement is taken into account, there remain infinitely many degrees of freedom in choosing a specific scheme of punishment. Based on a given scheme, one may vary costs for alternative values of $q$, provided (21) and (33) hold. As a limit, one may (in an imaginary situation) implement a $\Psi$*-scheme*, that is, a free-of-externalities scheme where costs of a given agent depend only on his own actions, *as if he were a representative of a homogenuous society of bureaucrats.* For such a scheme, costs equal $\psi(q)$, which justifies our title. This scheme would be our main reference point in the space of all the schemes (i.e., cost functions). The $\Psi$-scheme, firstly, reveals no external effects among bureaucrats and, secondly, serves as an imaginary limit of a process of a deliberate decrease in external effects *for all the other possible schemes simultaneously*, as reqired in (33). That is, our space of cost functions, as bounded by conditions (1) and (33), has a nice topological structure: it has one focal point with the property that all the curves from other points are approaching it if these curves are constructed by the gradual weakening of external effects. This is quite simple, for decreasing external effects means just approaching the diagonal, which is required to be unique for all the cost functions. The assertion below clarifies our intuition further.

**Assertion 3.1** *Whatever initial scheme $\varphi(q, q_{\rm av})$ has been chosen, the following inequalities hold which show that the free-of-externalities scheme is affected by an individual choice of an agent in the lowest degree, among all*

our considerations.

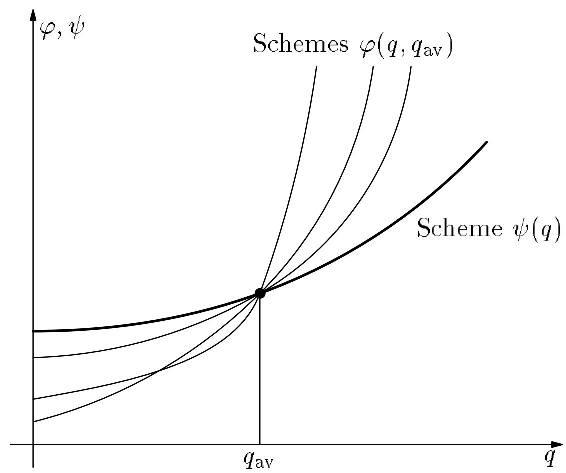


Figure 8: The diagonal scheme $\psi(q)$ is the limit case for all feasible schemes $\varphi(q, q_{\text{av}})$

*other feasible schemes (see Figure 8):*

$$\begin{aligned} q < q_{\text{av}} \quad &\Longrightarrow \quad \varphi(q, q_{\text{av}}) < \psi(q); \\ q > q_{\text{av}} \quad &\Longrightarrow \quad \varphi(q, q_{\text{av}}) > \psi(q). \end{aligned} \tag{34}$$

**Proof.** Recalling that the $\Psi$-scheme is the diagonal of any other scheme, hence, of a given scheme $\varphi$ (that is, $\psi(q) = \varphi(q, q)$), we can rewrite, for instance, the first inequality in the following way: $\varphi(q, q_{\text{av}}) < \varphi(q, q)$; provided $q < q_{\text{av}}$, this follows directly from decreasing $\varphi$ with respect to its second argument; see properties (21).

As a result, one can step-by-step transform a given scheme of punishment into the free-of-externalities scheme, holding properties (34) during the whole process of transformation. Putting it another way, among all the curves $G_\alpha(q, q_{\text{av}})|\alpha \in [0, 1]$ connecting a given scheme with the $\Psi$-scheme, there exists an (infinite-dimensional) family of curves characterized by the following properties:[18]

$$
\begin{array}{c}
\forall\alpha: \quad G_\alpha(q,q_{\mathrm{av}}) > \varphi(q,q_{\mathrm{av}}) \qquad \text{for} \quad 0<q<q_{\mathrm{av}}, \\
G_\alpha(q,q_{\mathrm{av}}) < \varphi(q,q_{\mathrm{av}}) \qquad \text{for} \quad q>q_{\mathrm{av}}, \\
G_\alpha(q,q_{\mathrm{av}}) = \varphi(q,q_{\mathrm{av}}) \qquad \text{for} \quad q=q_{\mathrm{av}}; \\
G_\alpha(q,q_{\mathrm{av}}) \quad \text{possesses properties of cost functions (21);} \\
\text{when} \quad \alpha=0 \quad G_\alpha(q,q_{\mathrm{av}}) \equiv \varphi(q,q_{\mathrm{av}}), \\
\text{when} \quad \alpha=1 \quad G_\alpha(q,q_{\mathrm{av}}) \equiv \varphi(q,q).
\end{array}
\tag{35}
$$

In the rest of the paper, I will analyse the behavior of the main characteristics of the equilibrium under consideration when a scheme of punishment (hence, a cost function) is being varied along such curves (see Figure 9). Parameter $\alpha$ will be interpreted as the intensity of the externalities in our system,[19] for along the way of a curve, that is, of this parameter's increase, the scheme of punishment approaches the free-of-externalities scheme.

## 3.2 The behavior at zero-point: two basic classes of families

We are now ready to turn to our main question. Consider a given scheme of punishment, $\varphi$, and some family (35) connecting it with the diagonal. We therefore deal with a parametrized family of economies of corruption, together with equilibria in these economies. We will call them *$\alpha$-economies* and *$\alpha$-equilibria*, respectively. We are interested in comparative statics with respect to parameter $\alpha$ introduced above. From now on, we use the terms *scheme of punishment* and *cost function* interchangibly; the term *diagonal*, or a *diagonal scheme* is always used to denote the (unique, common) free-of-externalities scheme.[20]

[18] Geometrically, these curves do not move in the opposite direction while approaching their (common) destination point.

[19] To be more precise, parameter $\alpha$ stands for the *weakness* of external effects.

[20] Recall that in reality this scheme could not be implemented; this fact should not confuse us, since we just refer to *infinitesimal* comparative statics (on the way along a curve approaching the focal point which is the diagonal scheme), that is, to comparative statics for small changes in $\alpha$.

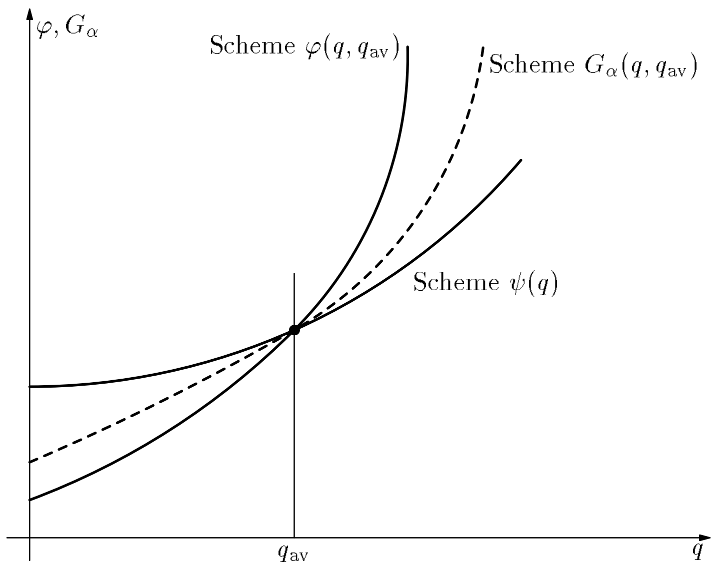


Figure 9: The family of cost functions $G_\alpha(q, q_{\mathrm{av}})$ connects an initial scheme $\varphi(q, q_{\mathrm{av}})$ and the diagonal one

Using notations introduced above, we can state $\alpha$-problem in the form which is nearly the same as (20) and (23), namely:

$$b(q, \theta) - G_\alpha(q, q_{\mathrm{av}}) \longrightarrow \max_{q \geq 0}, \tag{36}$$

$$\int q(\theta, q_{\mathrm{av}}) dF(\theta) = q_{\mathrm{av}}. \tag{37}$$

**Definition.** A bundle $(q^*_{\mathrm{av}}(\alpha), q^*(\alpha, \theta))$ is said to form $\alpha$-equilibrium if, for every $\theta$, a value $q = q^*(\alpha, \theta)$ solves problem (36), and the balance condition (37) holds.

All that was stated and proved in section 2.3 concerning the existence and the number of equilibria remains correct for $\alpha$-problems, taking into account properties (35).

The main question of our paper now could be expressed in the following manner: What is the effect of small changes in $\alpha$ on the average (equilibrium) level of corruption, $q^*_{\mathrm{av}}(\alpha)$, and on individual levels $q^*(\alpha, \theta)$?

Assumptions (35) about the behavior of a family of evolving cost functions include a variety of possible scenarios, from which we pick two groups which

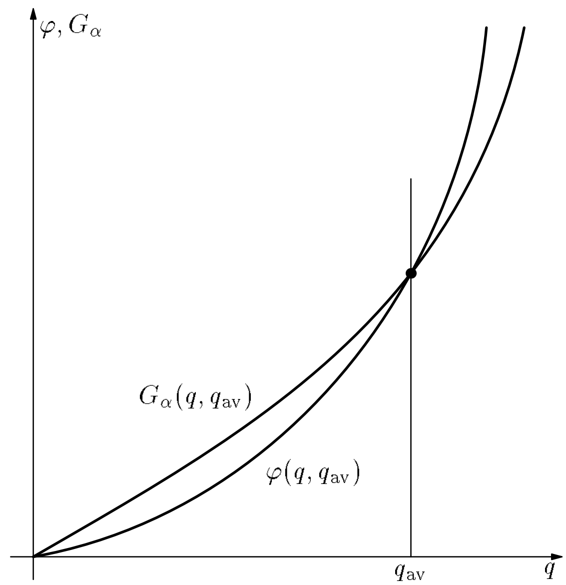


Figure 10: Family $G_\alpha(q, q_{\rm av})$ (the continuous case)

are probably the most important in applications.

The first group corresponds to the case of continuous cost functions (including zero-point). Assumptions (35) suggest the following graphical illustration (see Figure 10). Now we introduce an additional assumption (suggested by Figure 10) that the curvature of the initial scheme is higher than that of the diagonal one. Formally:

$$\forall q \qquad \frac{\partial^2 \varphi}{\partial q^2}(q, q_{\rm av}) > \psi''(q). \tag{38}$$

Among all the families connecting $\varphi$ with the diagonal, we will analyse only those for which the curvature (with respect to $q$) decreases, as $\alpha$ rises (just as illustrated in Figure 10).[21] In other words, we require that a second derivative with respect to $q$ decreases as $\alpha$ increases, for any given $q$. This case will be studied in section 3.3.

According to the second group of scenarios, all the cost functions of a family, $G_\alpha(q, q_{\rm av})$ have a jump in zero. We distinguish two principal cases:

[21]Again, these families *approach the focal point smoothly* in a geometric sense, being considered in the (infinitely-dimentioned) space of all schemes of punishment.

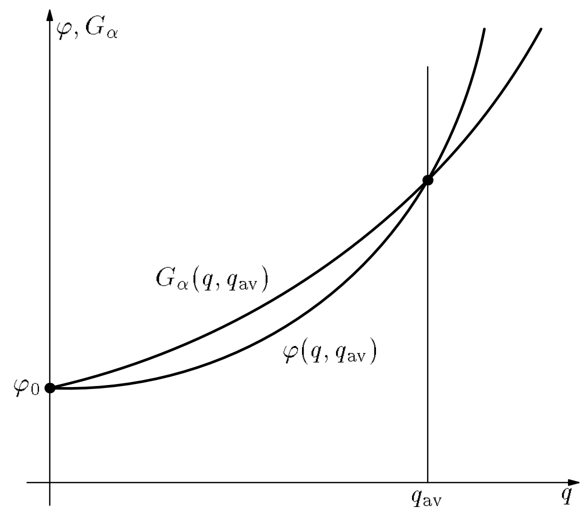


Figure 11: Family $G_\alpha(q, q_{\mathbf{av}})$ (the case of a unique jump in zero)

when the jump is unique for all cost functions; and when it rises with $\alpha$ (it apparently could not decrease, as far as properties (35) hold, since

$$\varphi_0 = \lim_{q\to 0} \varphi(q, q_{\rm av}) \le \lim_{q\to 0} G_\alpha(q, q_{\rm av}) = G_{\alpha 0}, \tag{39}$$

where we have denoted the value of a jump of a typical cost function in our family by $G_{\alpha 0}$). Figures 11 and 12 correspond to the two possibilities, respectively.

The possibility of a unique jump seems to be not a good approximation of reality. Indeed, its applicability suggests probably that fixed costs from corruption come *solely* from moral factors, even free of a *common feelings* argument. We dispence of this possibility in favor of concentrating on the last case when a jump in zero is an increasing function of parameter $\alpha$. Above that, we deal with those (initial) schemes only for which *marginal* costs from corruption exceed marginal costs of the diagonal scheme, for all $q$. Such a restriction is suggested by Figure 12. Formally:

$$\forall q \ge 0 \qquad \frac{\partial \varphi}{\partial q}(q, q_{\rm av}) > \psi'(q) = \frac{\partial \varphi}{\partial q}(q, q) + \frac{\partial \varphi}{\partial q_{\rm av}}(q, q). \tag{40}$$

Note that, when calculating marginal costs of the diagonal scheme, one should take the *full derivative*, not a partial one, with respect to the first ar-

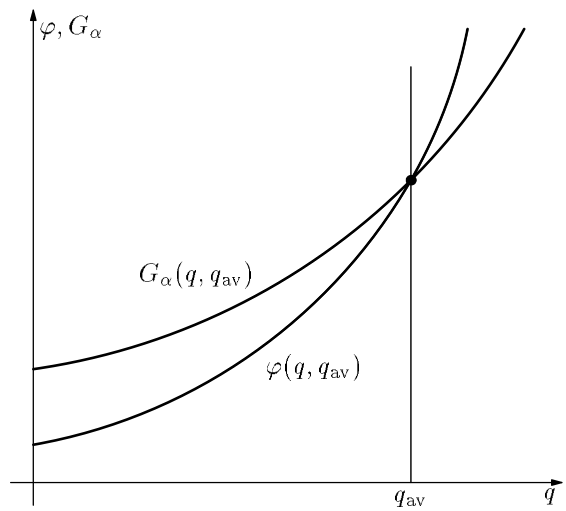


Figure 12: Family $G_\alpha(q, q_{\mathbf{av}})$ (the jump varies with $\alpha$)

gument (the latter corresponds to marginal costs of the *initial* scheme, when $q_{\mathrm{av}} = q$). The same should be kept in mind when looking at inequality (38).

In accordance with what we have done in the continuous case, here we assume that as $\alpha$ increases, marginal costs of functions in a family monotonically decrease for any given $q$ from values of marginal costs of the initial scheme to those of the diagonal one.[22]. We analyse the case of a jump in zero in section 3.4.

## 3.3 Comparative Statics: the continuous case

In the current section we assume that all the functions in family (35) are well defined and continuous with respect to $q$ over the full half-axis $q \geq 0$, including zero. One can check that properties (35), together with our additional requirement about second derivatives, result in the following list

[22]We again refer to a geometric interpretation in the space of all feasible cost functions: the class of families chosen satisfies the smoothness condition, in a sense.

of assumptions:

$$\begin{array}{c} G_\alpha(q, q_{\mathrm{av}}) \quad \text{are well defined and continuous over} \quad [0, +\infty); \\ \forall\alpha: \quad G_\alpha(0, q_{\mathrm{av}}) \equiv 0, \qquad G_\alpha(q_{\mathrm{av}}, q_{\mathrm{av}}) \equiv \varphi(q_{\mathrm{av}}, q_{\mathrm{av}}); \\ \dfrac{\partial G_\alpha}{\partial q}(0, q_{\mathrm{av}}) \quad \text{increases with respect to}\ \alpha; \\ \forall q \quad \dfrac{\partial^2 G_\alpha}{\partial q^2}(q, q_{\mathrm{av}}) \quad \text{decreases with respect to} \quad \alpha; \\ G_0(q, q_{\mathrm{av}}) \equiv \varphi(q, q_{\mathrm{av}}), \qquad G_1(q, q_{\mathrm{av}}) \equiv \psi(q). \end{array} \tag{41}$$

Our task is to analyse how the solution to the problem described by (36) and (37) changes when $\alpha$ increases. Recall Theorem 4: in every stable equilibrium, the long-run effect of a change in parameters has the same sign but a *higher* amplitude than the short-run effect. The latter was expressed by the reaction function, $\Psi(q_{\mathrm{av}})$). Hence, to answer our main question, we should analyse the behavior of the reaction function $\Psi(\alpha, q_{\mathrm{av}})$ (depending on two arguments, from now on), in response to changes in argument $\alpha$.

Recall the formula for the reaction function:

$$\Psi(\alpha, q_{\mathrm{av}}) = \int q(\alpha, \theta, q_{\mathrm{av}}) dF(\theta), \tag{42}$$

where $q(\alpha, \theta, q_{\mathrm{av}})$ is an instant response of an agent with type $\theta$ to an increased $\alpha$, in other words, a solution to problem (36), for fixed $q_{\mathrm{av}}$.

Thus, after all, we are interested in individual behavior of agents-bureaucrats of various types, in response to an increased $\alpha$. In the continuous case currently being analysed, (36) is equivalent to the following first-order condition:

$$\begin{array}{c} \dfrac{\partial b}{\partial q}(q, \theta) \leq \dfrac{\partial G_\alpha}{\partial q}(q, q_{\mathrm{av}}), \\ \text{with strict equality for} \quad q > 0. \end{array} \tag{43}$$

Let us recall that the first-order conditions for the initial problem are:

$$\begin{array}{c} \dfrac{\partial b}{\partial q}(q, \theta) \leq \dfrac{\partial \varphi}{\partial q}(q, q_{\mathrm{av}}), \\ \text{with strict equality for} \quad q > 0. \end{array} \tag{44}$$

The left hand side has not changed since corruption opportunities remained the same. As for the right hand side, the most convenient way to analyse its behavior is graphically. Let us turn to Figure 13 which illustrates

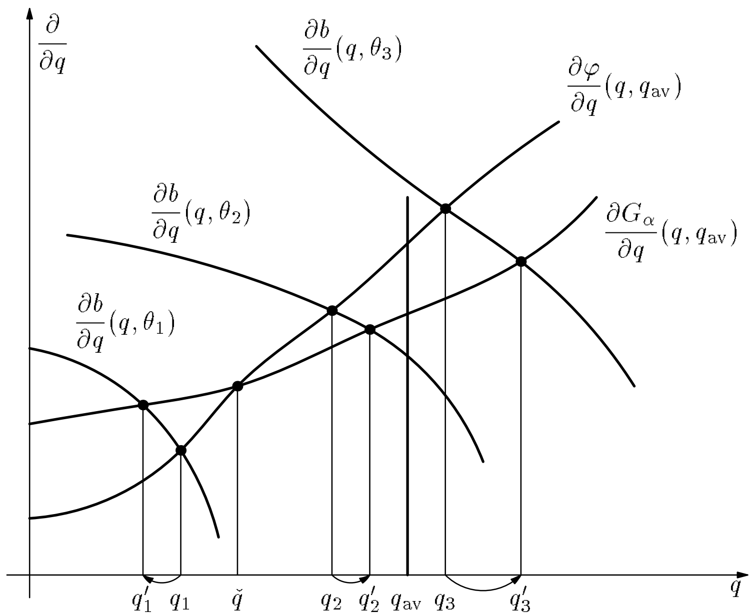


Figure 13: The effect of increasing $\alpha$ (the continuous case, $\theta_1 < \theta_2 < \theta_3$)

the first-order conditions. A family of downward slopping curves denote marginal benefits from corruption, for alternative types of agents, $\theta$, whereas two upward slopping ones are marginal cost curves, for zero and non-zero $\alpha$ corresponding to the initial problem and to one of the family problems, respectively. Assumptions (41) imply that

$$\frac{\partial G_\alpha}{\partial q}(q_{\rm av}, q_{\rm av}) < \frac{\partial \varphi}{\partial q}(q_{\rm av}, q_{\rm av}). \tag{45}$$

We can definitely say that this is the key observation in this paper. Also, one can easily see that there exists a unique point of intersection of the two marginal cost curves. Denoting it by $\breve{q}(\alpha)$, we have

$$\frac{\partial G_\alpha}{\partial q}(\breve{q}(\alpha), q_{\rm av}) = \frac{\partial \varphi}{\partial q}(\breve{q}(\alpha), q_{\rm av}), \tag{46}$$

and $0 < \breve{q}(\alpha) < q_{\rm av}$. After these preparations, the following assertion becomes obvious.[23]

[23]An important task is to study the behavior of $\breve{q}(\alpha)$. Here we omit this subject, for it

**Assertion 3.2** *When $\alpha$ goes up:*

- *agents whose initial choice used to be located to the left of $\breve{q}$ decrease their corruption level;*
- *agents with $q^*(\theta) = 0$ choose honest behavior (as before);*
- *agents whose initial choice was higher than $\breve{q}$ increase their degree of involvement in corruption.*[24]

*Formally,*

$$\begin{cases} q^*(\theta) < \breve{q} & \Longrightarrow \quad q(\alpha, \theta, q^*_{\mathrm{av}}) < q^*(\theta) \\ q^*(\theta) > \breve{q} & \Longrightarrow \quad q(\alpha, \theta, q^*_{\mathrm{av}}) > q^*(\theta) \end{cases}. \tag{47}$$

Intuitively, agents with comparatively good corruption opportunities will increase their level of involvement in corrupt relations because as $\alpha$ increases, the burden of punishment mitigates. On the contrary, agents with bad opportunities may find it profitable to decrease their level of involvement, for now they are subject to tighter persecution.[25] Therefore, what one can unambiguously state about the short-run effect is the increased differentiation in agents' choices, which is a useful observation.

As for the short-run utility of agents, one can state the following.

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is a part of supplementary research concerning the infinitesimal analysis and goes beyond the goals of the current paper.

[24]Indeed, assume, for instance, that $q^*(\theta) < \breve{q}$. Then, the RHS of (43) exceeds the LHS in the point of this agent's initial choice. But, as $q$ increases, the RHS of (43) increases as well, whereas the LHS decreases. Hence, a new choice is located to the left of the initial one. The opposite is true when $q^*(\theta) > \breve{q}$.

[25]However, note that there always exist two effects of increased involvement in corruption: the income effect (higher amount of bribes collected), and the punishment effect. Since these two have opposite directions, the sign of a resulting effect is a priori ambiguous.

**Assertion 3.3** *When $\alpha$ increases infinitesimally, the utility of agents whose initial choice was below the average level of corruption falls, whereas the utility of other agents rises.*

The precise formulation and its correct proof are omitted here, for they are part of supplementary research mentioned above (devoted to infinitesimal analysis). Intuitively, the direction of a change in the utility of a given agent coincides for small changes in parameter $\alpha$ with the direction of the change in the cost function (according to the Envelope Theorem). Therefore, having properties (35) in mind, agents with $q < q_{\mathrm{av}}$ will suffer from varying the scheme of punishment; others, on the contrary, will benefit.

Up to now, we have investigated the individual behavior of agents quite thoroughly. But our main question concerns the average level of corruption: how does it respond to changes in $\alpha$? As a priori different agents respond in an opposite manner, we cannot predict the behavior of the average level of corruption unambiguously. Nevertheless, note that $\breve{q} < q_{\mathrm{av}}$, hence, one can expect that *more than half* of the agents increase their level of involvement in corruption. In order to make this statement precise and formal, we should focus on one of two possible lines of research (resulting in two alternative formulations of sufficient conditions for the average level of corruption to increase, as a result of an increased $\alpha$, stated below).

**Theorem 5** *If every agent's initial choice exceeded $q \geq \breve{q}$, that is, if $\forall\theta\, q(\theta, q_{\mathrm{av}}) \geq \breve{q}$, then the average corruption level, $q_{\mathrm{av}}$, increases unambiguously as $\alpha$ rises.*

**Proof.** In such a situation, all the agents increase their corruption levels in the short-run, hence, the $\Psi$-curve shifts upwards. As we already know, this leads to an increased average level of corruption.

Note that this assertion admits a trivial but important consequence: in the case when all the agents are *identical*, the average corruption level increases with diminishing external effects arising from implementing a new, more individualistic, scheme of punishment. Moreover, even if agents differ from each other, but their actual *choices* are identical in the initial equilibrium, assertion 5 still holds.

The analysis conducted concerns the case of an *interior* equilibrium only (i.e., the one with a strictly positive corruption level). As far as the corruption-free equilibrium is concerned, we just mention here that it may become unstable, as a result of a decentralizing policy. Unfortunately, we have no opportunity to dwell upon the study of the corruption-free equilibrium in this paper.

In order to formulate alternative sufficient conditions, let us introduce the notion of an increment choice function. This function shows the absolute change in the agent's choice, depending on his type. Denote the increment choice function by $\triangle q(\alpha,\theta)$. Formally, we have

$$\triangle q(\alpha,\theta) = q(\alpha,\theta,q^*_{\rm av}) - q^*(\theta), \tag{48}$$

where, as before, $q^*(\theta)$ stands for the choice of an agent with type $\theta$, see section 2.3. Besides, we simplify the formulation of our problem a bit, assuming that agents are distributed *uniformly* over a set of possible types.[26] Now we are ready to specify the conditions sufficient for the average level of corruption to unambiguously increase, with $\alpha$ increased.

**Theorem 6** *The average level of corruption increases if, first, function $\triangle q$ is convex with respect to $\theta$, and, second, the median agent's (i.e., the agent with type $\theta_{\rm av}$) choice is at least high as $\breve{q}$.*

For **Proof**, see Appendix. Also, an alternative formulation of this theorem is discussed there which deals with another way of reparametrizing agents in a system: according to it, an agent's type is his initial choice, $q^*(\theta)$. Such an interpretation allows for the discarding of the second condition of the theorem, though at the expence of giving up clear intuition (note, however, that this second condition is not too binding: once we knew that $\breve{q} < q_{\rm av}$, we naturally expect the choice of the medium agent not to fall much shorter of the average level of corruption).

Convexity-like conditions are quite theoretically convenient. Intuitively, they seem to reflect a big differentiation in corruption opportunities. Assuming this interpretation to be true, one can conclude that the theorem of an

[26]The case of an absolutely continuous distribution of agents could be reduced to the one considered here automatically: all that changes is the functional form of $b(q,\theta)$. One can easily check that under such a reparametrization, the list of requirements (18) remains the same.

increased average corruption level holds in two diametrically opposite cases: the one with almost identical corruption opportunities, and the other with high differentiation. Unfortunately, we cannot state this theorem to hold unconditionally, but counter-examples should look quite odd, so to speak. Generally, one can expect that, provided the cost function is continuous, the average level of corruption will increase in response to weakening external effects.

Let us now turn to the second main case where the cost function has a jump in zero. We will find our results need to be corrected substantially.

### 3.4 Comparative Statics: the case of a jump in zero

Again, before turning to the formal analysis of families of cost functions (35) exhibiting discontinuity in zero, it is useful to summarize all the assumptions made about them:

$$\begin{gathered} G_\alpha(q, q_{\mathrm{av}}) \quad \text{are well-defined and continuous over} \quad (0, +\infty); \\ G_\alpha(q_{\mathrm{av}}, q_{\mathrm{av}}) \equiv \varphi(q_{\mathrm{av}}, q_{\mathrm{av}}); \qquad \forall q \quad \frac{\partial G_\alpha}{\partial q}(q, q_{\mathrm{av}}) \quad \text{decreases with respect to} \quad \alpha; \\ G_0(q, q_{\mathrm{av}}) \equiv \varphi(q, q_{\mathrm{av}}); \qquad G_1(q, q_{\mathrm{av}}) \equiv \psi(q). \end{gathered} \tag{49}$$

In contrast with the analysis in the previous section, first-order conditions (43) no longer characterize the solution correctly. Similary as in section 2.3, we introduce *double terminology*. Namely, let $q_{\mathrm{av}}^*$ and $\bar{q}^*(\theta)$ denote the equilibrium and the agents' choice function in the initial problem, whereas $q^*(\theta)$ stands for the solution to first-order conditions (22), that is, the *intermediate* choice; Similary, let $q(\alpha, \theta, q_{\mathrm{av}}^*)$ denote the parametrized family of solutions to the first-order conditions, for alternative $\alpha$ and $q_{\mathrm{av}} = q_{\mathrm{av}}^*$:[27]

$$\begin{gathered} \frac{\partial b}{\partial q}(q, \theta) \leq \frac{\partial G_\alpha}{\partial q}(q, q_{\mathrm{av}}), \\ \text{with strict equality for} \quad q > 0. \end{gathered} \tag{50}$$

Finally, $\bar{q}(\alpha, \theta, q_{\mathrm{av}}^*)$ will denote the actual choice of the agent with type $\theta$, that is, the solution to problem (36) with $q_{\mathrm{av}} = q_{\mathrm{av}}^*$ (it coincided with $q(\alpha, \theta, q_{\mathrm{av}}^*)$,

[27] As before, we analyse here a short-run effect, having in mind that it only accumulates, in the long-run.

in the continuous case). Also, we will use abbrevated notations $q(\alpha, \theta)$ and $\bar{q}(\alpha, \theta)$, respectively.

Let us now turn to the utilities of the agents. In order to analyse it we again introduce double notations. That is, let us call by the *resulting utility* of a given agent his utility when he chooses $q(\alpha, \theta, q^*_{\text{av}})$, instead of $\bar{q}(\alpha, \theta, q^*_{\text{av}})$:

$$U(\alpha, \theta) = b(q(\alpha, \theta), \theta) - G_\alpha(q(\alpha, \theta), q^*_{\text{av}}) - G_0(\alpha), \tag{51}$$

where $G_0(\alpha)$ is the value of a jump in zero which, according to our assumptions (49), increases in $\alpha$. And we will refer to the result of the actual (full) maximization (including the question of whether to be involed in corruption ever) by the *actual utility*. The latter implies choosing $\bar{q}(\alpha, \theta, q^*_{\text{av}})$ (let us denote the actual utility by $\bar{U}(\alpha, \theta)$). Formally:

$$\bar{U}(\alpha, \theta) = U(\alpha, \theta)_+ = \begin{cases} U(\alpha, \theta), & \text{if } U(\alpha, \theta) \geq 0 \\ 0, & \text{else} \end{cases}. \tag{52}$$

Also, we will use notations $U^*(\theta)$ and $\bar{U^*}(\theta)$, respectively for the resulting and the actual utilities of an agent with type $\theta$ in the initial equilibrium.

Firstly, we study the individual behavior of the agents, in response to an increased $\alpha$. The following assertion is the starting point of our analysis.

**Assertion 3.4** *When $\alpha$ increases, the intermediate choice of all the agents, $q(\alpha, \theta, q^*_{\text{av}})$ increases (as long as it was greater than zero; otherwise it could either increase, or remain to be zero).*

**The proof** almost repeats that of Assertion 3.1, and is omitted here. The essential thing is that the marginal cost curve monotonically shifts down in our case, as $\alpha$ increases (see Figure 14). Hence, every agent's choice shifts to the right.

As for the actual choice function, its formal description is

$$\bar{q}(\alpha, \theta, q^*_{\text{av}}) = \begin{cases} q(\alpha, \theta, q^*_{\text{av}}), & \text{if } U(\alpha, \theta) \geq 0 \\ 0, & \text{else} \end{cases}. \tag{53}$$

We saw in section 2.3 that this is equivalent to the following representation (see Assertion 2.1):

$$\bar{q}(\alpha, \theta, q^*_{\text{av}}) = \begin{cases} q(\alpha, \theta, q^*_{\text{av}}), & \text{if } \theta \geq \bar{\theta}(\alpha) \\ 0, & \text{if } \theta < \bar{\theta}(\alpha) \end{cases}, \tag{54}$$

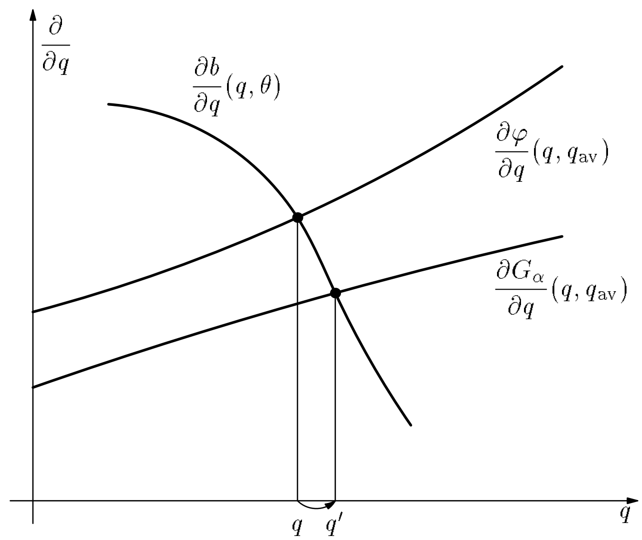


Figure 14: The effect of an increased $\alpha$ on the intermediate choices of the agents (the case of a jump in zero)

where $\bar{\theta}(\alpha)$ satisfies the equation $U(\alpha, \bar{\theta}(\alpha)) = 0$. Therefore, one should carefully analyse the resulting utility.

**Assertion 3.5** *The incremental resulting utility of the agents is an increasing function of their type. The resulting utility of the agents, whose choice falls short of $q_{\text{av}}$, decreases monotonically when $\alpha$ increases.*

A more careful formalization and proof of Assertion 3.5 are omitted here, for they are a part of the supplementary research to this paper mentioned above. As for the cut-off value $\bar{\theta}(\alpha)$, its behavior is a priory ambiguous. One can infer from Assertion 3.5 that if $\bar{q}(\alpha) = q(\alpha, \bar{\theta}(\alpha)) < q_{\text{av}}$ then $\bar{\theta}(\alpha)$ should unambiguously increase. In the opposite case, the cut-off value may well decrease, especially if $\bar{q}(\alpha) = q(\alpha, \bar{\theta}(\alpha)) \gg q_{\text{av}}$, which corresponds to the case when there is a comparatively small fraction of highly corrupt bureaucrats, while the others do not take bribes at all. Below we state, without a proof, the precise result.

**Assertion 3.6** *A derivative of $\bar{\theta}(\alpha)$ with respect to $\alpha$ takes the form*

$$\bar{\theta}'(\alpha) = \frac{\dfrac{\partial G}{\partial \alpha}(\bar{q}(\alpha)) + \dfrac{\partial G_0}{\partial \alpha}}{\dfrac{\partial b}{\partial \theta}(\bar{q}(\alpha))}. \tag{55}$$

Switching to the main question (that of the dynamics of the average level of corruption), we recall the definition of the reaction function (42), and rewrite it, using (54):

$$\Psi(\alpha, q^*_{\text{av}}) = \int \bar{q}(\alpha, \theta, q^*_{\text{av}})dF(\theta) = \int_{\bar{\theta}} q(\alpha, \theta, q^*_{\text{av}})dF(\theta). \tag{56}$$

Figures 15 and 16 characterize the cases of increasing and decreasing cut-off level $\bar{\theta}(\alpha)$, respectively. Figure 15 suggests the formulation of the basic theorem proposing sufficient conditions for the increase in the average level of corruption, in response to weakening external effects, in a system where cost functions of the family have a discontinuity (a jump) in zero. Simultaneously, we get sufficient conditions for the increase of differentiation in individual corrupt choices.

**Theorem 7** *The average level of corruption unambiguously increases if the cut-off level $\bar{\theta}(\alpha)$ falls with $\alpha$. The differentiation in individual corruption levels unambiguously increases in the opposite case (i.e., when the cut-off level increases with $\alpha$).*[28]

**The proof** is trivial: in the first case, corruption recruites new agents while incumbents make corrupt deals more intensively. One can see that the situation is ambiguous if the condition of this theorem fails to hold. In such a case, however, *differentiation* in the agents' choices increases for sure: less bureaucrats become involved in corrupt relations, but those who continue to be involved are doing it more intensively.

At this stage, we end up the analysis of the model constructed, although many questions remain unanswered which require ongoing research. In the

[28]It may well occur that *both* differentiation and the average level of corruption rise. Indeed, we presented only *sufficient*, but not *necessary*, conditions in this theorem.

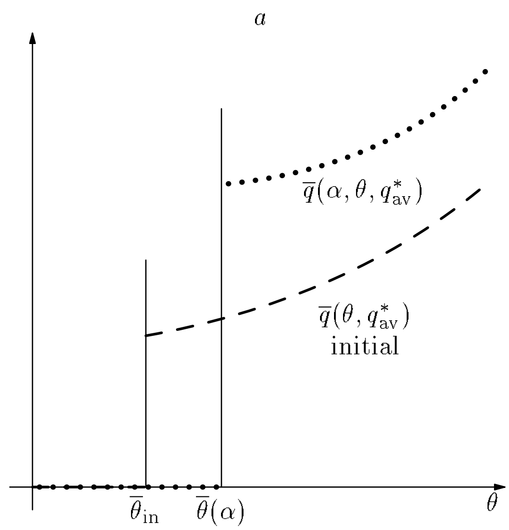


Figure 15: The effect of an increased $\alpha$ on the behavior of the actual agents' choice function, $\bar{q}(\theta, q^*_{\mathrm{av}})$. The case of increasing cut-off level $\bar{\theta}(\alpha)$

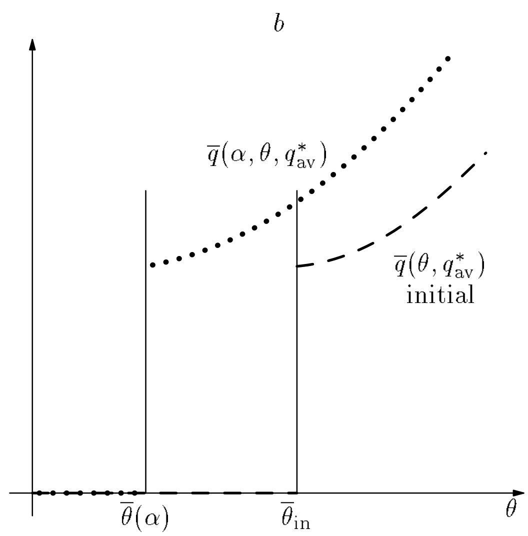


Figure 16: The effect of an increased $\alpha$ on the behavior of the actual agents' choice function, $\bar{q}(\theta, q^*_{\mathrm{av}})$. The case of decreasing cut-off level $\bar{\theta}(\alpha)$

next section, I summarize the main conclusions and provide appropriate interpretations in terms of real anti-corruption policy, thus characterizing their practical value.

# 4 Conclusions and policy recommendations

Now it is time to summarize the basic results of the conducted research. In this paper, the model of corruption in production economies was constructed which takes into account two kinds of factors of corruption, namely what we called (following Polterovich (1998)) *fundamental* and *societal* ones. Factors of the first group are derived from economic and institutional characteristics of the system under study, while the second-type factors stand for strategic interaction among agents (here: bureaucrats), in other words, they reflect *external effects* inherent in such systems. A huge body of published research on the subject of corruption is devoted to analysing the effects from either factors of the first group or of the second one, but not both factors simultaneously (see the literature review in the next section). However, factors of both types are intimately connected with each other, and our model, being a unifying framework for both types of factors, is an addition to understanding the nature and essence of corrupt relations (at least, in production processes). Our assumption that a bureaucrat chooses the *degree* of involvement in corrupt relations by solving a corresponding optimization problem where costs from corruption depend not only on his choice but also on the average level was discussed also in Sah (1991), but from different positions (there, crime rates were focused upon); the simple dilemma *to be or not to be a criminal* was posed first by Becker (1968). Still, almost no research has encorporated, up to this moment, these external effects into the production environment. This was the contribution of this paper.

The model was analysed thoroughly. The existence of equilibria was established, and we found that in a typical situation there is a corruption-free stable equilibrium and another stable equilibrium with a high level of corruption. Focusing our attention on the latter, we revealed the so-called *accumulation effect*: long-run shifts in the average level of corruption, in response to changes in various parameters, multiply shifts in the short-run perspective.

In addition, agents' individual choices were characterized. It turned out that the function specifying agents' choices is continuous when the cost function is continuous over the closed interval $[0, +\infty)$, and reveals a stepwise behavior if the cost function has a jump in zero.

The main goal, however, was to compare alternative schemes of punishment in the framework of the model constructed. Restricted to a fixed budget, one is still free to choose one or another pattern of corruption deterrence.

A fixed budget requirement was interpreted in terms of our model's parameters, though not precisely. A focal (imaginary) scheme was postulated as the one where external effects are ignored by the policymaker when considering how to allocate the resources at hand, and the main question was whether the policymaker should try to approximate such a scheme (some publications refer to externalities as a driving force for corruption; see, e.g., Polterovich (1998)). There is no exact answer. Nevertheless, definite features were established. We recall some of them here, trying not to use mathematical but policymaker's language.

When it is felt that there are no fixed costs from corrupt activity so that *almost* honest behavior generates *almost* no costs, the policymaker is faced with the problem that accounting for externalities in a lesser degree leads to higher differentiation in agents' choices. Moreover, such a bias results in *favoring* more corrupt agents while threatening *moderate* ones. As for the average level of corruption, it probably only increases. Thus, one is suggested to move in the opposite direction exploring external effects in order to get a higher degree of uniformity in different agents' choices.

If, on the contrary, a fixed cost part is expected to arise, the policymaker, while weakening his reference to the average level of corruption, confronts higher bribes and a higher degree of illegal activity *from those who decide to be corrupt as before*. If, in addition, new agents enter the pool of corruptioneers, the policymaker will definitely get a higher average level of corruption. In case when many agents refrain from corruption as a result of such a policy, the average level may decrease, but in this case differentiation increases substantially.

Thus, choosing the specific form of a punishment scheme requires taking all these circumstances into account, while analysing the situation at hand. All that was established in this paper may help the policymaker to form his position towards fighting the corruption taking place under his jurisdiction.

# A review of literature

# and the place of our research

The substantial analysis in the current paper is based on the classification of factors of corruption which was is adopted from Polterovich (1998). It was emphasized that it is the simultaneous work of both types of factors that determine the nature of corrupt relations in production. And we noted that except for Acemoglu, Verdier (1997), most of the papers devoted to corruption elaborate on just one type of factor. Nevertheless, many insights were first gained from these investigations, and it is high time to pay for these ideas, with special attention to research on corruption and similar topics. It is convenient to divide a whole amount of research into three main groupes.

**Class A. Rent-seeking and corruption: accounting for fundamental factors**

The first to systematically study corruption from positions of fundamental and organizational factors was Rose-Ackerman (1975). Actually, these workings contain the first attempt to model a market for corruption services precisely, though in a quite general form. Moral factors and the factor of possible punishment are taken into account. A zone of feasible bribes is characterized, and a proper mechanism of making corrupt decisions by a bureaucrat is revealed. Organizational factors are also accounted for, in the form of a specific hierarchical structure of different bureaucrats in the system and how this structure affects the level of corruption activity. It is shown that under some circumstances, corruption is not dangerous since one expects it not to be realized in practice, while under other conditions it could become a widespread phenomenon. Alternative anti-corruption measures are analysed in the paper. What makes the paper highly relevant for our approach is that it is also based on corruption relations in the production process, namely, firms are competing for a Government project to fulfill. In the book by Rose-Ackerman (1978), the material is systemized, different forms of corruption are classified and subjected to a detailed analysis.

Another seminal paper providing a comparative analysis of alternative anti-corruption measures accounting for *industrial organization of corrupt relations* is by Shleifer, Vishny (1993). In contrast with our research, a red tape phenomenon is considered when bribes are being extorted for the fulfillment of bureaucrats' direct obligations. Ignoring the possibility of punishment, the authors characterize three different forms of bureaucratic structure and stretch them in a strict line with respect to social harm: the best structure for the society is when bureaucrats are competing for clients; the second-best is the unique (monopolistic) bribes collection, like the Mafia or the Communist Party of the Soviet Union; the worst is when bribes are being extorted independently and chaotically by different bureaucratic branches (for example, post-soviet Russia, at least during 1992-1996). Besides, negative consequences of the necessary secrecy of markets for corrupt services are discussed, as well as the cathastrophic consequences for research and development.

Guriev (1998) conducted an interesting research on the importance of the hierarchical bureaucratic structure in determining bribe levels and the scale of red tape phenomenon.

In the current paper, the focus is made on the corruption phenomenon, while other kinds of a *deviating* behavior deserve careful study. Two years ago, we (together with Prof. Polishchuk) analysed sociological aspects of rent-seeking phenomenon. A paper by Polishchuk and Savvateev (1997) somehow inspired the current research, being to a certain degree consistent with its methodology (see also Savvateev (1997)). We studied a generalised version of rent-seeking, following the ideology of the seminal works of Tullock (1980) and Skaperdas (1996). It was assumed that there are two kinds of activities accessible for agents in an economy: production and redistribution. It is a common knowledge that a fixed share of what is being produced in a system is subject to redistribution. The production function was of the simpliest possible (one-factor, called a *resource*,) type and we assumed that the same resource is being used in redistribution. Production function is characterized by decreasing returns while the redistribution sector reveals constant returns to scale. A corresponding coefficient is determined so that the market for expropriation clears. Of course, a simple banning of redistribution is socially desirable since it enriches the economy as a whole (all the resources are then used in production and are not wasted away in redistribution activity), but it turns out that provided some conditions hold a part of agents (moreover, more than a *half* of agents) may not favor such a reform, for it decreases their *individual* welfare. Glaser (1993) also demonstrates (though within an entirelly different approach) how a candidate whose program promises not to decrease the amount of rents in the economy could well be elected. As a result, the reform is blocked or postponed. We also stated and proved conditions under which the reform, on the contrary, is a Pareto-improvement. Putting it briefly, these conditions require that returns to scale in production are not decreasing too sharply.

Polterovich (1998) used a similar model to analyse corruption equilibria, with respect to their stability and social efficiency. It turns out that sometimes producers vote for corruption: it helps them to get rid of an inefficient tax burden. A striking result is that under some circumstances a *vast majority* of producers would prefer to deal with moderately corrupt officials.

Ericson (1983) analyses the soviet shadow economy and comes to the conclusion that its presence sometimes guarantees a Pareto-improvement over an imaginary entirely planned regime. In fact, bribes clear the market, and this motivation is quite popular for overcontrolled economic organizations.

**B. Corruption: A Societal Approach**

During the last decade, an approach to explaining corruption by societal factors has been quite popular, i.e., that the main source of stability and pervasive expansion of corruption could be found in stable patterns of behavior, properly examined over decades or even centuries and seen in the nature of interconnections and positive external effects prevailing in the corrupt environment. We already cited Polterovich (1998) which in fact is a starting point for our research. The main conclusion of the author was that a switch from bad to good equilibrium may be the result of implementing a short-run but extremely tough anti-corruption policy.

Lui (1986) comes to similar conclusions in a framework of the overlapping generations model. Tirole (1996) is even more pessimistic. He analysed corruption from *collective reputation* positions when two types of agents interact in the economy, and all the agents of the first type are partially identified with each other, from the point of view of the agents of the second type. The author emphasizes the importance of personal gains from deviating behavior in forming a stable long-run corruption equilibrium, despite the fact that collective reputation matters. Based on a very clear and intuitive model, he shows that sometimes an instant stimulus to be dishonest could fully determine the ongoing story, generating a stable *corrupt* pattern of behavior. Alas, even a very tough anti-corruption policy implemented during a short period of time may not bring a change in this socially harmful behavioral pattern. After relaxing penalties, corruption would return again to the system. As a matter of fact, it could be quite inertial phenomena.

On the contrary, Biccieri and Rovelli (1995) demonstrated how, under certain conditions, a stable corruption equilibrium may suddenly loose its stability and eventually be replaced by a corruption-free one, provided there is a certain fraction of agents who *never* take bribes.

Finally, a paper by Acemoglu and Verdier (1997) deserves special attention since this is the only one to take both factors of corruption into account. Authors model an economy with two different productions: the first one is with *good* technology (for it produces positive externalities), and the other one is with *bad*. In a competitive equilibrium, all producers choose bad technology (for reasons similar to those resulting in the famous Prisoner's Dilemma). In order to correct the situation, the government hires a certain amount of bureaucrats whose direct obligation is to check whether firms choose good technology. However, they have stimula to collect bribes instead in exchange for delivering wrong information to the government concerning a technology actually chosen. Numerical externalities are taken explicitly into account while modelling the monitoring process — the more deviating firms, the less probably a given deviator will be caught. Assuming heterogenuity of bureaucrats, it is proved that even the optimal organization suggests an excessively large staff of officials, excessively high salaries for bureaucrats in the public sector, and a certain *positive* level of corruption.

**C. Crime, punishment and tax evasion**

Another line of research intimately connected to ours is devoted to the general *crime and punishment problem* or to (probably) its most popular special case, i.e., tax evasion. Models of this sort also focus attention on the problem of choosing the optimal scheme of punishment which either prevents criminal behavior or equates marginal (social) costs with marginal costs of fighting it, i.e., the problem of choosing and enforcing the *optimal level of* criminal behavior of a given kind. A seminal work on these issues is by Becker (1968). A somewhat different view is enlighten in Sah (1991), and we based our analysis here substantially on his ideas. Out of a huge body of tax evasion literature, let us mention Vasin and Panova (2000), and also Chander and Wilde (1992) and Sanchez and Sobel (1993).

# Appendix. Proofs of the main theorems

**Proof of Theorem 1.**

Let us write down the Lagrangian for problem (5):

$$\begin{gathered}\Lambda(q_{ij},\lambda,\mu_j) \;=\; \sum_j b_{ij}(q_{ij}) - \lambda\cdot\left(\sum_j q_{ij} - q_i\right) - \sum_j \mu_j\cdot q_{ij},\\ \mu_j \le 0, \quad \lambda \ge 0.\end{gathered} \tag{57}$$

Recalling that $b_{ij}$ is convex, one can conclude that the first-order conditions are necessary and sufficient for this problem:

$$\begin{gathered}b'_{ij}(q_{ij}) - \lambda - \mu_j = 0, \quad j = 1,...,N_i;\\ \mu_j\cdot q_{ij} = 0, \quad j = 1,...,N_i;\\ \lambda\cdot\left(\sum_j q_{ij} - q_i\right) = 0.\end{gathered} \tag{58}$$

Actually it means that either $\lambda = 0$ or for all positive $q_{ij}$

$$b'_{ij}(q_{ij}) \equiv \lambda, \tag{59}$$

whereas for all other indices $j \quad b'_{ij}(0) < \lambda$.

In the first case we have $\forall j \quad b'_{ij}(q_{ij}) = 0$, that is all the choices $q_{ij}$ are satiation points for corresponding functions $b_{ij}$. Such a situation could be an equilibrium only if the value of $q_i$ chosen by a bureaucrat is excessively high: $q_i \ge \sum_j q_{ij}$, for all feasible levels of illegal activities.[29]

In the second case, for sufficiently small values of $\lambda$ we have $\sum_j q_{ij}(\lambda) > q_i$. Our task is to find such $\lambda$ that $\sum_j q_{ij}(\lambda) = q_i$, where $q_{ij}(\lambda)$ denotes a solution to (59) or zero. It is easy to check that functions $q_{ij}(\lambda)$ are continuous and nonincreasing (this is again due to properties of functions $b_{ij}$, see (18), hence, their sum as well is continuous and monotonic. For $\lambda > \max_j(b'_{ij}(0))$ this sum becomes zero. Therefore, there exist a unique $\lambda$ satisfying conditions (58), hence, being (together with $q_{ij}(\lambda)$) a solution to problem (5).

The intuition of what is going on is straightforward. Here $\lambda$ represents nothing else but a *shadow price* of illegal services for a bureaucrat. In the equilibrium, price is determined in a manner that guarantees the equality of the overall amount of illegal services supplied by this bureaucrat to the

[29]It is clear from the equilibrium analysis of the basic model of interacting bureaucrats that this case never happens: no one bureaucrat would choose such $q_i$ unless the costs from corruption are independent of his choice.

initially chosen value of $q_i$. In this way, producers who are not rich enough to support price $\lambda$ will be ignored by the bureaucrat, i.e., their $q_{ij} = 0$.

From now on, $\lambda(q_i)$ denotes a shadow price of a service, $q_{ij}(q_i)$ stands for amounts of illegal services to producers, and the set of all $j$ for which $q_{ij} \neq 0$ is denoted by $J$. In addition, let us exclude from consideration the trivial and irrelevant case of $\lambda = 0$.

We now turn to the study of function $b_i(q_i) = \sum\limits_{j\in J} b_{ij}(q_{ij}(q_i))$. Differentiating with respect to $q_i$ one gets

$$\begin{aligned} \frac{\partial b_i}{\partial q_i} &= \sum_{j\in J} \frac{\partial b_{ij}(q_{ij}(q_i))}{\partial q_i} = \sum_{j\in J} \left(\frac{\partial b_{ij}}{\partial q_{ij}}(q_{ij}(q_i)) \cdot \frac{\partial q_{ij}(q_i)}{\partial q_i}\right) = \\ &= \lambda \cdot \sum_{j\in J} \frac{\partial q_{ij}(q_i)}{\partial q_i} = \lambda \cdot \frac{\partial \sum\limits_{j\in J} q_{ij}(q_i)}{\partial q_i} = \lambda \cdot \frac{\partial q_i}{\partial q_i} = \lambda \end{aligned} \tag{60}$$

using the first-order conditions. Therefore, first of all, $b_i'(q_i) > 0$, and second, $b_i'(0) = \max_j(b_{ij}'(0))$ since one can easily check that when $q_i \to 0$, we have $\lambda \to max_j(b_{ij}'(0))$.

The next step is the behavior of the second derivative, $b_i''(q_i)$. Due to (60), one has $b_i''(q_i) = \lambda'(q_i)$, whereas the latter could be found by differentiating the following equality with respect to $q_i$:

$$\sum_j q_{ij}(\lambda) \equiv q_i. \tag{61}$$

Taking derivatives, we get

$$1 \equiv \left(\sum_j q_{ij}'(\lambda)\right) \cdot \lambda'(q_i). \tag{62}$$

Since derivatives of $q_{ij}(\lambda)$ are nonpositive, so is the derivative of their sum. Hence, $\lambda'(q_i) \leq 0$. As a result, the concavity of the function $b_i(q_i)$ should hold.

The boundedness of function $b_i$ follows from the boundedness of functions $b_{ij}$. The proof of Theorem 1 is complete.

A verbal (intuitive) description is given below. Function $b_i(q_i)$ starts from zero with a speed equal to the maximum of the derivatives of functions $b_{ij}$ at zero-point, actually coinciding with the function on which this maximum is reached. As $q_i$ increases, a sum determining $b_i$ "absorbs" new items which reflects the involvement of new producers (with lower production capacities) in a group illegally supplied by the bureaucrat. Eventually, satiation is reached, due to the boundedness of the overall monetary stock of all the producers.

**Proof of Theorem 6**.

The proof is based on Jensen's Inequality which is written below (for convex functions).

**Jensen's Inequality**. *Let $f$ be a convex function. Then, for an arbitrary chosen integrating set $A$, one has*

$$\int_A f(\theta)d\theta \geq f\left(\int_A \theta d\theta\right). \tag{63}$$

Now we can write, using (42), (48), (63), (23), and the second condition of the theorem, the following chain of equalities and inequalities:

$$\begin{aligned}\Psi(q^*_{\text{av}}) - q^*_{\text{av}} = \textstyle\int (q(\alpha,\theta,q_{\text{av}}) - q^*(\theta))\, dF(\theta) = \int \triangle q(\alpha,\theta) dF(\theta) \geq \\ \geq \triangle q\,(\alpha, \textstyle\int \theta dF(\theta)) = \triangle q(\alpha,\theta_{\text{av}}) \geq 0.\end{aligned} \tag{64}$$

The proof is complete.

The importance of theorem 6 consists in the fact that its conditions are expressed in terms of the model's data, not in terms of a specific equilibrium chosen. It is done at the expense of making an additional assumption concerning the medium agent's initial choice. Below we demonstrate how one can exchange this additional assumption for giving up invariant terms.

We had proved earlier (see (28)) that the agents' choice function is nondecreasing. In the regular case, one can make a monotonic reparametrization such that a new parameter would be the agent's choice $q^*(\theta)$ in the initial equilibrium. Then, first, different equilibria generate different reparametrizations, and second, the distribution's *density* of the agents with respect to their types, $f(\theta) = F'(\theta)$, would turn out not to be uniform (and also will depend on a specific equilibrium choice). However, if we managed to establish the concavity of the increment choice function as a function of $q^*$ *with respect to the corresponding density*$F'(q^*)$,[30] we could be confident that Theorem 6 continues to hold since its fulfillment is again based on Jensen's Inequality. Summing up, we have the following theorem.

**Theorem 8** *If the increment choice function $\triangle q(\alpha, q^*)$ is convex in $q^*$ with respect to the density of the distribution of agents over their $q^*$-types, $F'(q^*)$, then the average level of corruption increases with parameter $\alpha$.*

[30]We omit here all the pecularities of the formal definition of this notion. We simply skip its definition, instead accounting entirely on intuitive understanding.

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# REFERENCES

Acemoglu D., Verdier T. The Choice Between Market Failures and Corruption. Draft, August 1997.

Bardhan P. Corruption And Development: A Review of Issues. Journal of Economic Literature, Vol. XXXV (September 1997), pp. 1320-1346.

Basu S., Li D. Corruption in Transition. Revised draft, May, 1998.

Becker G. Crime And Punishment: An Economic Approach. Journal of Political Economy, 76 (March/April 1968), pp. 169-217.

Biccieri C., Rovelli C. Evolution and Revolution. Rationality and Society, Vol. 7, No. 2, April 1995, pp. 201-224.

Chander P., Wilde L. Corruption in Tax Administration. Journal of Public Economics, 49 (1992), pp. 333-349.

Ericson R. On An Allocative Role of the Soviet Second Economy. In: P. Desai (ed). Marxism, Central Planning, and the Soviet Economy. Cambridge, M.A.: the MIT Press, 1983.

Glaser A. On The Incentives to Establish And Play Political Rent-Seeking Games. Public Choice 75, 1993, pp. 139-148.

Guriev S. Bribing Thy Boss's Boss: A Model of Top-Level Corruption. New Economic School and Russian European Center for Economic Policy, WP, November 1998.

Lui F. A Dynamic Model of Corruption Deterrence. JPE 31 (1986), pages 215-236.

Mas-Colell A., Whinston M., Green J. Microeconomic Theory. Oxford University Press, New-York, 1995.

Polishchuk L., Savvateev A. Spontaneous Emergence of Property Rights: A Critical Analysis. Paper presented at the conference "Transforming Government in Transition Economies," September 17, Moscow. M.: New Economic School, 1997.

Polterovich V. Corruption And The Tax Policy. CEMI, February 1998, Moscow (unpublished).

Polterovich V. Faktori korruptsii. Ekonomika i Matematicheskie Metodi, issue 3, vol. 34 (1998).

Rose-Ackerman S. "The Economics of Corruption". Journal of Public Economics 4 (February 1975), pp.187-203.

Rose-Ackerman S. Corruption: A Study in Political Economy. New York, Academic Press, Inc., 1978.

Sah R. Social Osmosis And Patterns of Crime. Journal of Political Economy, Vol. 99, no.6, 1991, pp.1272-1295.

Sanchez I., Sobel J. Hierarchical design and enforcement of income tax policies. Journal of Public Economics, 50 (1993), pp. 345-369.

Satarov G., Levin M., Tsirik M. Rossiya i korruptsiya: kto kogo? A Report in Rossiiskaya Gazeta, February 19 (1998).

Savvateev A. Production And Rent-seeking Behavior. Working Paper BSP/98/001, NES, CEMI, 1998.

Shleifer A., Vishny R. Corruption. Quarterly Journal of Economics, August, 1993, pp. 519-617.

Skaperdas S. Contest Success Functions. Economic Theory, Vol.7, 1996, pp. 283-290.

Tirole J. A Theory of Collective Reputations With Applications to The Persistence of Corruption And to Firm Quality. Institut d'Economie Industrielle, Toulouse, MIT and Ceras, Paris, 1996.

Tullock G. Efficient Rent-Seeking. In Toward a Theory of the Rent-seeking Society, edited by James M. Buchanan, Robert D. Tollison, and Gordon Tullock. College Station: Texas AM University Press, 1980.

Vasin A., Panova E. Tax Collection and Corruption in Fiscal Bodies. EERC Working Paper No 99/10 (2000).

Wei Sh.-J. Why Is Corruption So Much More Taxing Than Tax? Arbitrariness Kills. NBER Working Paper Series, Working Paper 6255, November 1997.